Name: ______________________________ ________ / 12

- There are 12 points possible on this proficiency: **One point per problem. No partial credit.**
- A passing score is 10/12.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Be sure to include constants of integration when appropriate.
- Circle your final answer.

Compute the following integrals.

1. $\int_1^2 \frac{4-3x^4+x^8}{x^5}\,dx = \int_1^2 (4x^{-5}-3x^{-1}+x^3)dx$

OK here

$= -x^{-4} - 3\ln|x| + \frac{1}{4}x^4\Big]_1^2 = \left(\frac{-1}{2^4} - 3\ln(2) + \frac{2^4}{4}\right) - \left(\frac{-1}{1^4} - 3\ln(1) + \frac{1}{4}\cdot 1^4\right)$

$= -\frac{1}{16} - 3\ln(2) + 4 - \left(-1 - 0 + \frac{1}{4}\right) = -\frac{1}{16} - 3\ln(2) + 5 - \frac{1}{4} = 4\frac{11}{16} - 3\ln(2)$

2. $\int_0^1 (\cos(3x) - e^{-x})\,dx = \frac{1}{3}\sin(3x) + e^{-x}\Big]_0^1 = \left(\frac{1}{3}\sin(3) + e^{-3}\right) - \left(\frac{1}{3}\sin(0) + e^0\right)$

$= \frac{1}{3}\sin(3) + \frac{1}{e^3} - 1$

3. $\int \frac{\sec^2(x)}{\tan(x)-2}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln|\tan x - 2| + C$

Let $u = \tan x - 2$

$du = \sec^2 x\, dx$

4. $\int \cos^2(2x)\sin(2x)\,dx$ $= -\frac{1}{2}\int u^2\,du = -\frac{1}{6}u^3 + C$

let $u = \cos(2x)$

$du = -2\sin(2x)\,dx$

$-\frac{1}{2}\,du = \sin(2x)\,dx$

$= -\frac{1}{6}(\cos(2x))^3 + C$

5. $\int \frac{x}{\sqrt{4-x}}\,dx$ $= \int x(4-x)^{-1/2}\,dx = -\int (4-u)u^{-1/2}\,du = -\int (4u^{-1/2} - u^{1/2})\,du$

let $u = 4-x$

$du = -dx$

$-du = dx$

$x = 4-u$

$= \int (u^{1/2} - 4u^{-1/2})\,du = \frac{2}{3}u^{3/2} - 8u^{1/2} + C$

$= \frac{2}{3}(4-x)^{3/2} - 8(4-x)^{1/2} + C$

6. $\int \frac{x}{1-x^2} + \frac{2}{1+x^2}\,dx$ $= -\frac{1}{2}\ln|1-x^2| + 2\arctan(x) + C$

let $u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}\,du = x\,dx$

7. $\int \frac{e^{x^{1/3}}}{x^{2/3}}\,dx$ $= 3\int e^u du = 3e^u + C$

Let $u = x^{1/3}$

$du = \frac{1}{3}x^{-2/3}dx$

$3du = \frac{dx}{x^{2/3}}$

$= 3e^{x^{1/3}} + C$

8. $\int (3x+3)(x+1)\,dx$ $= 3\int (x+1)(x+1)\,dx = 3\int (x^2+2x+1)\,dx$

$= 3\left(\frac{1}{3}x^3 + x^2 + x\right) + C$

$= x^3 + 3x^2 + 3x + C$

9. $\int x^2 \sin(1-x^3)\,dx$ $= -\frac{1}{3}\int \sin u \, du = \frac{1}{3}\cos u + C$

Let $u = 1-x^3$

$du = -3x^2 dx$

$-\frac{1}{3}du = x^2 dx$

$= \frac{1}{3}\cos(1-x^3) + C$

10. $\int \sqrt{x}(x^2 + \frac{2}{x^2})\, dx$ $= \int x^{\frac{1}{2}}(x^2 + 2x^{-2})\, dx = \int (x^{5/2} + 2x^{-3/2}) dx$

$= \frac{2}{7} x^{7/2} + 2 \cdot (-2) x^{-1/2} + C$

$= \frac{2}{7} x^{7/2} - 4x^{-1/2} + C$

11. $\int \left(\frac{e^x + x^3}{\sqrt{3}}\right) dx$ $= \frac{+1}{\sqrt{3}} \int (e^x + x^3) dx = \frac{+1}{\sqrt{3}} \left(e^x + \frac{1}{4}x^4\right) + C$

12. $\int \frac{4 - 3(\ln x)^2}{x}\, dx$ $= \int \frac{4}{x}\, dx - \int \frac{3(\ln x)^2}{x}\, dx$

$= 4\ln|x| - (\ln x)^3 + C$