

Name: \_\_\_\_\_

### Rules:

You have 90 minutes to complete this midterm.

Partial credit will be awarded, but you must show your work.

Calculators, notes and books are not allowed.

Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	12	
2	5	
3	9	
4	10	
5	10	
6	24	
7	10	
8	10	
9	10	
Extra Credit	5	
Total	100	

1. (12 points) Compute and simplify the improper integrals, or show that they diverge. Use correct limit notation.

(a)  $\int_2^{\infty} \frac{dx}{x(\ln(x))^2}$

(b)  $\int_0^3 \frac{1}{x^{4/3}} dx$

2. (5 points) Does the series  $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$  converge or diverge? Show your work including naming any test you use. (Hint: You may use the previous problem though you don't have to. )

3. (9 points) Consider the infinite series  $-\frac{1}{2 \cdot 1} + \frac{1}{2 \cdot 2} - \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 4} - \frac{1}{2 \cdot 5} + \frac{1}{2 \cdot 6} - \dots$

(a) Write the series using sigma or summation notation. (That is, write the series using  $\sum$  notation.)

(b) Compute and simplify  $S_1$ ,  $S_2$ , and  $S_3$  the first three terms in the sequence partial sums of the series.

4. (10 points) Consider the infinite series  $\sum_{n=0}^{\infty} \frac{(-3)^{n+1}}{10^n}$ .

(a) Explain why the series converges.

(b) Determine the sum of the series. Write your answer as a simplified fraction.

5. (10 points) Show that the series  $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{2n+1}}$  is conditionally convergent.

Note that you must show that the series converges **and** that it is not absolutely convergent.

A complete answer will include (i) the name of the test(s) you are using, (ii) a clear application of the test (or tests), and (iii) an explicit explanation of what conclusion(s) you are drawing.

6. (24 points) Do the following series converge or diverge? Show your work, including naming any test you use.

(a) 
$$\sum_{n=0}^{\infty} \frac{2n-1}{5n+1}$$

(b) 
$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$$

$$(c) \sum_{n=1}^{\infty} \frac{\sin(n)}{n^{5/3}}$$

$$(d) \sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^n}$$

7. (10 points) Use  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$  to find power series representations centered at  $a = 0$  for each function below.

(a)  $g(x) = \frac{x}{1-3x}$

(b)  $h(x) = \frac{1}{(1+x)^2}$  (Hint: Differentiate an appropriate function.)

8. (10 points) Write the Taylor series for  $y = e^{-2x}$  centered at  $a = 1$ .

9. (10 points) Find the interval of convergence of the following power series.

(a) 
$$\sum_{n=0}^{\infty} \frac{(x-3)^n}{(n+1)!}$$

(b) 
$$\sum_{n=1}^{\infty} \frac{x^n}{n8^n}$$

**Extra Credit** (5 points) The Taylor series for  $f(x) = \sin(x)$  centered at  $a = 0$  is  $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$ .

1. Find  $p_3(x)$ , the 3rd Taylor polynomial, and use it to estimate  $\sin(1)$ .
2. Show that this estimate is within 0.005 of the exact value.