Math F252 Final Exam Fall 2025

Name: _____

Rules:

You have 120 minutes to complete this midterm.

Partial credit will be awarded, but you must show your work.

Calculators are not allowed.

One hand-written sheet of notes is allowed.

Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	15	
2	6	
3	6	
4	10	
5	20	
6	6	
7	10	
8	10	
9	6	
10	5	
11	6	
Extra Credit	5	
Total	100	

1. (5 pts each) Evaluate the indefinite integrals.

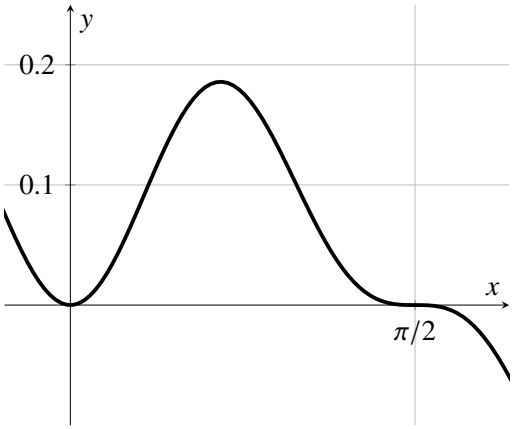
   (a) $\displaystyle\int \frac{1}{x^2\sqrt{x^2-4}}dx$

   (b) $\displaystyle\int \frac{1}{x^2-9}dx$

(c) $\int (4x - xe^x)dx$

2. (6 pts) The shaft of a bird feather has density function $\boxed{\rho = \frac{2}{5}\arctan x}$ grams per meter on the interval from $x = 0$ m to $x = 1$ m. Find the **mass** of the shaft. **Include units with your answer.**

3. (6 pts) The region R is bounded by $y = \sin^2(x)\cos^3(x)$ and the x-axis between $x = 0$ and $x = \pi/2$. Find the **area** of the region R.



4. (10 pts) The region R is bounded by $y = e^x$, $y = 3$, and the y-axis.

   (a) Sketch the region R. Label the curves and all points of intersection of the curves.

   (b) Using either the disks/washers or cylindrical shells method, **set up an integral** to compute the volume generated when R is rotated around the x**-axis**. **State which method you are using.** You do not need to evaluate the integral.

   (c) Using either the disks/washers or cylindrical shells method, **set up an integral** to compute find the volume generated when A is rotated around the y**-axis. State which method you are using.** You do not need to evaluate the integral.

5. (5 pts each) Determine whether each series below converges or diverges. Name the test you use and justify your conclusion. (This means show your work!)

(a) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{5n^2+1}\right)$ Test: _____ Converge or Diverge

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2+3n}{3+4n^{3/2}}$ Test: _____ Converge or Diverge

(c) $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$ Test: _____ Converge or Diverge

(d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{3n+2}}$ Test: _____ Converge or Diverge

6. (6 pts) Use the Integral Test to determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$ converges or diverges.

7. (5 pts each) For each power series below, determine the interval of convergence.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{n5^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!(x+4)^n}{7^{2n}}$

8. (10 pts) Recall that the Maclaurin series $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$.

   (a) Write $p_4(x)$, the 4th-degree Maclaurin **polynomial** for $\cos(x)$.

   (b) Find the Maclaurin series for $f(x) = x\cos(2x)$. Simplify your answer.

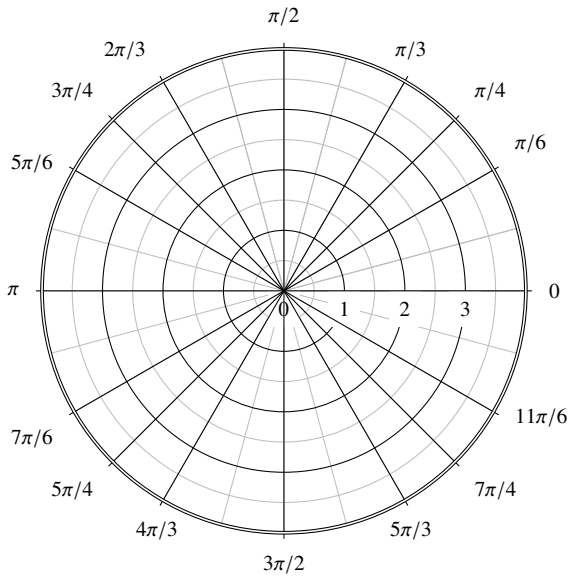
   (c) Find the Maclaurin series for $G(x) = \int_0^x \cos(\sqrt{t})\, dt$.

9. (6 pts) Consider the curve $x(t) = t^2 - 3t + 1$, $y(t) = e^t$.

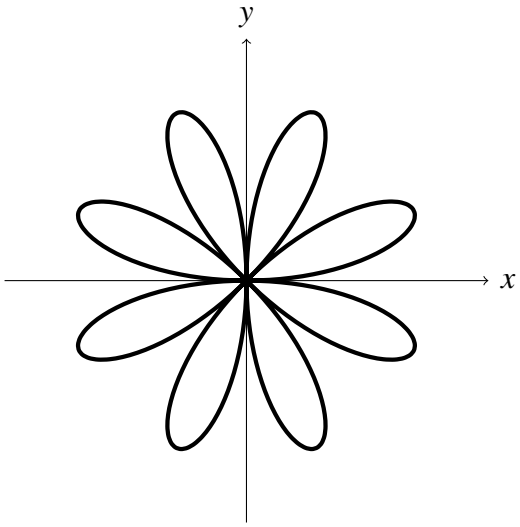
   (a) Determine the slope of the curve at the point $(1, 1)$.

   (b) Determine the points where the tangent line is horizontal or vertical, or state that none exist.

10. (5 pts) Make a careful sketch of the polar curve $r = 1 + 2\sin(\theta)$. Label at least 4 points.



11. (6 pts) Compute the area enclosed by the polar curve $r = 3\sin(4\theta)$. The graph of the curve is shown below.



Extra Credit (5 points) Let $f(x) = \sum_{n=0}^{\infty} \binom{\frac{1}{2}}{n} \frac{x^n}{3^n}$.

(a) Determine the first 4 terms in the series. Simplify the coefficients.

(b) The series converges when $x = 2$. Determine the sum.