Math F252 Midterm 2 Fall 2025

Name: _____

Rules:

You have 90 minutes to complete this midterm.

Partial credit will be awarded, but you must show your work.

Calculators are not allowed.

One hand-written sheet of notes is allowed.

Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	8	
2	8	
3	12	
4	8	
5	24	
6	8	
7	12	
8	12	
9	8	
Extra Credit	5	
Total	100	

1. (8 pts) Consider the sequence $S = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \cdots\right\}$.

   (a) (4 pts) Find a formula for the general term a_n of the sequence assuming that the pattern of the first few terms continues.

   (b) (2 pts) Does this sequence converge? Justify your conclusion.

   (c) (2 pts) Does series $\sum_{n=1}^{\infty} a_n$, with terms from the sequence S, converge? Justify your conclusion.

2. (8 pts) Determine if the integral below converges or diverges. Evaluate the integral if it converges. To earn full points, a solution must contain clear complete work and correct use of notation.

$$\int_0^4 \frac{1}{(4-x)^{2/3}}\, dx$$

3. (12 pts) For each **convergent** series below, determine its sum.

(a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3}{n^4}-\frac{3}{(n+1)^4}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-5)^{n-1}}{2^{3n}}$.

4. (8 pts) Use the **Integral Test** to determine if the series $\sum_{n=1}^{\infty} ne^{-n^2}$ converges or diverges. (You do not have to verify that the Intergral Test applies.)

5. (6 pts each) For each series below, show whether the series converges or diverges using an appropriate test. **State the test you use.**

   (a) $\sum_{n=1}^{\infty} \frac{n^6}{2^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(n+3)!}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \ln\left(\frac{1}{n}\right)$

(d) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt[3]{n+2}}$

6. (8 pts) Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n^3}{5+n^5}$ is **absolutely convergent, conditionally convergent, or divergent.**

7. (12 pts) Determine the radius of convergence, R, and the interval of convergence for each power series below.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{5n+4}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

8. (12 pts)

(a) Find a power series representation for the function $f(x) = \dfrac{x}{1-6x}$.

(b) Determine the radius of convergence and interval of convergence for the power series in part (a).

(c) Use your answer from part (a) to find a power series representation for $f'(x)$.

9. (8 pts) Find the Taylor series for $f(x) = \frac{1}{x}$ at $a = 3$. Your answer should be simplified.

Extra Credit (5 pts) Determine the convergence of the two series below.

a. $\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)^n$

b. $\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)$