Math F252 Final Spring 2026

Name: ____________________________________

Rules:

You have **2 hours** to complete this midterm.

Partial credit will be awarded, but you must show your work.

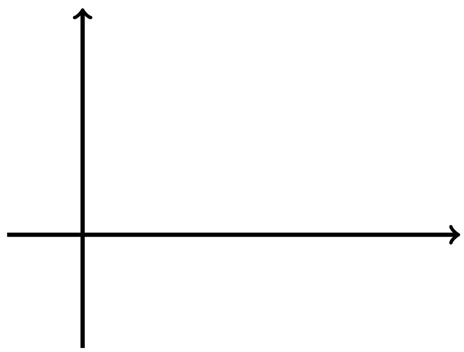
Calculators and books are not allowed. You may have **one whole sheet** of letter paper, 2-sided, with notes.

Place a box around your FINAL ANSWER to each question.

Turn off anything that might go beep during the exam. Good luck!

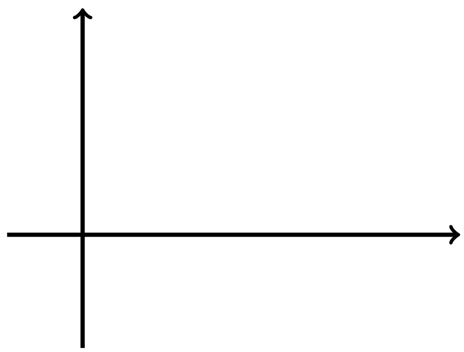
Problem	Possible	Score
1	6	
2	6	
3	13	
4	12	
5	6	
6	6	
7	15	
8	12	
9	9	
10	11	
11	14	
12	9	
13	6	
Extra Credit A	*2*	
Extra Credit B	*2*	
Total	125	

1. (*6 pts*) Sketch the area under the curve $f(x) = \ln x$ between $x = 1$ and $x = e$. Find this area by evaluating an integral.



2. (*6 pts*) Evaluate (compute the value of) the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}}$.

3. **(a)** (*3 pts*) On the axes below, sketch the region R in the first quadrant bounded by $y = \cos(x)$, $y = 0$, and $x = 0$.



(b) (*5 pts*) Use an integral to find the volume of the solid obtained by rotating R about the x-axis.

(c) (*5 pts*) Set up, but do not evaluate, an integral for the surface area of the solid in part **(b)**.

4. Evaluate the indefinite integrals below.

(a) (*6 pts*) $\int \cos^5(\theta)\, d\theta$

(b) (*6 pts*) $\int \frac{1}{(4+x^2)^{3/2}}\, dx$

5. (*6 pts*) A spring with a relaxed length of 2 meters requires 3 Newtons force to stretch to a length of 2.1 meters. How much work would it take to stretch the spring from 2 meters to 2.4 meters?

6. (*6 pts*) Find the Maclaurin series of $\sinh x = \dfrac{e^x - e^{-x}}{2}$. Express your answer as **one series**.

7. (*5 pts each*) Do the following series converge or diverge? Show your work; **state the name** of any test you use.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{2n+1}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{4^{n+1}}{n^n}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$

8. (*6 pts each*) For each power series, determine the **interval** of convergence.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{8(n+1)!}$

9. Let $f(x) = \dfrac{1}{1+x}$.

(a) (*4 pts*) Find a formula for $f^{(n)}(x)$, the nth derivative of $f(x)$.

(b) (*5 pts*) Find the Taylor series for $f(x)$ centered at $a = 1$. Your answer should be reasonably simplified.

10. Consider the curve defined by the parametric equations $x = \ln(t), \quad y = (t-1)^2.$

(a) (*4 pts*) Find the equation of the tangent line at $t = 2$.

(b) (*3 pts*) For $t > 0$, determine the points on the curve—give the x, y coordinates of these points—at which the tangent line is horizontal.

(c) (*4 pts*) Set up but do not evaluate an integral for the length of the curve from $t = 1$ to $t = 2$.

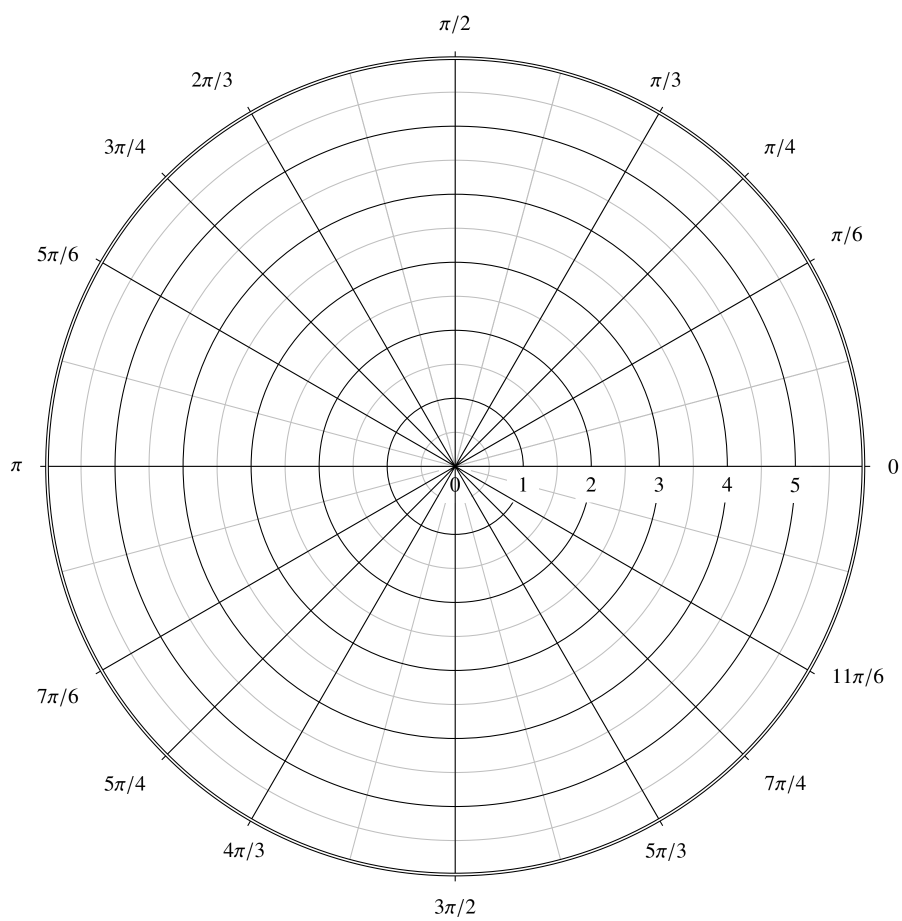
11. (a) (*4 pts*) Find the Maclaurin series for e^{-x^2}. (**Hint.** Start from a known Maclaurin series.)

(b) (*5 pts*) Compute an infinite series for the integral $\int_0^3 e^{-x^2}\,dx$.

(c) (*5 pts*) Show that the infinite series in part **(b)** converges. **State the name** of any test you use.

12. **(a)** (*4 pts*) Convert the polar equation $r = \cos\theta$ to rectangular form.

(b) (*5 pts*) Sketch the polar curve $r = 3 + 3\sin\theta$.



13. (*6 pts*) Evaluate and simplify the indefinite integral $\int e^{2x} \sin x \, dx$.

Extra Credit A (easier). (*2 pts*) If you compute the millionth partial sum of the infinite series in problem **7 (a)**, about how many digits would you expect to be accurate?

Extra Credit B (harder). (*2 pts*) Evaluate the integral in problem **3 (c)**.

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You may find the following **trigonometric formulas** useful. Other formulas, not listed here, should be in your memory, or you can derive them from the ones here.

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$$

$$\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$$

$$\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$$