CORRECTED 9/2

Math 252: Quiz 1 SOLUTIONS **1 September, 2022**

Name: ______________________ ________ / 24

30 minutes maximum. 24 points possible; each part is worth 2 points. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

1. [12 points] Compute the derivatives of the following functions.

a constant

a. $f(x) = e^2 x^{1/2} + 2e^x + \sqrt{9}$

$$f'(x) = e^2 \cdot \tfrac{1}{2} x^{-1/2} + 2e^x = \frac{e^2}{2\sqrt{x}} + 2e^x$$

b. $f(x) = \ln\left(\cos(x^3) - 4x^7\right)$

CORRECTED

$$f'(x) = \frac{1}{\cos(x^3) - 4x^7} \cdot \left(-\sin(x^3)3x^2 - 28x^6\right)$$

$$= -\frac{3x^2\sin(x^3) + 28x^6}{\cos(x^3) - 4x^7}$$

c. $h(x) = \sin(kx^2 - 5)$ where k is a constant

$$h'(x) = \cos(kx^2 - 5)(2kx)$$

$$= 2kx\cos(kx^2 - 5)$$

d. $f(x) = \sec(xe^x)$

$$f'(x) = \sec(xe^x)\tan(xe^x)(1\cdot e^x + x\cdot e^x)$$

$$= e^x(1+x)\sec(xe^x)\tan(xe^x)$$

e. $y = \dfrac{\cos(2x)}{x^5+\pi}$

CORRECTED

$$\frac{dy}{dx} = \frac{-2\sin(2x)(x^5+\pi) - \cos(2x)(5x^4)}{(x^5+\pi)^2}$$

product here

product here

f. Find $\dfrac{dy}{dx}$ if $e^y\cos(x) = xy+1$. You must solve for $\dfrac{dy}{dx}$.

$$e^y\frac{dy}{dx}\cos(x) + e^y(-\sin(x)) = 1\cdot y + x\cdot\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(e^y\cos(x) - x\right) = y + e^y\sin(x)$$

$$\frac{dy}{dx} = \frac{y+e^y\sin(x)}{e^y\cos(x)-x}$$

"$u = 1+x$" does not work here

2. [12 points] Compute the following antiderivatives (indefinite integrals) and definite integrals. Remember that antiderivatives need a "$+C$".

a. $\displaystyle\int \frac{(1+x)^2}{2x}\,dx$

$$= \int \frac{1+2x+x^2}{2x}\,dx = \int \frac{1}{2}\frac{1}{x} + 1 + \frac{1}{2}x\,dx$$

$$= \frac{1}{2}\ln|x| + x + \frac{x^2}{4} + c$$

b. $\displaystyle\int (x-1)\,e^{\left((x-1)^2\right)}\,dx$

$$u = (x-1)^2$$
$$du = 2(x-1)\,dx$$
$$\frac{du}{2} = (x-1)\,dx$$

$$= \int e^u \frac{du}{2}$$

$$= \frac{1}{2}e^u + c$$

$$= \frac{1}{2}e^{\left((x-1)^2\right)} + c$$

c. $\displaystyle\int_0^{\pi} 5e^x + 3\sin(x)\,dx$

$$= 5e^x - 3\cos(x)\Big]_0^{\pi}$$

$$= \left(5e^{\pi} - 3(-1)\right) - \left(5e^0 - 3\cdot(1)\right)$$

$$= 5e^{\pi} + 3 - 5 + 3 = 5e^{\pi} + 1$$

d. $\int x\sqrt{x+5}\,dx$

$$= \int (u-5)\sqrt{u}\,du$$

this form is better because you can distribute

$$\left[\begin{array}{l} u = x+5 \Leftrightarrow x = u-5 \\ du = dx \end{array}\right]$$

key part

$$= \int u^{3/2} - 5u^{1/2}\,du$$

$$= \frac{2}{5}u^{5/2} - 5\cdot\frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{5}(x+5)^{5/2} - \frac{10}{3}(x+5)^{3/2} + C$$

e. $\int \frac{\cos(\ln x)}{x}\,dx$

$$= \int \cos(u)\,du \qquad \left[\begin{array}{l} u = \ln x \\ du = \frac{1}{x}dx \end{array}\right]$$

$$= \sin(u) + C = \sin(\ln x) + C$$

f. $\int \frac{\sec^2(x)}{\tan^2(x)}\,dx$

$$= \int \frac{du}{u^2} = \int u^{-2}\,du \qquad \left[\begin{array}{l} u = \tan x \\ du = \sec^2 x\,dx \end{array}\right]$$

$$= -u^{-1} + C = -(\tan x)^{-1} + C$$

$$= -\cot x + C$$

don't write "$\tan^{-1} x$" because it is ambiguous