Math 252: Quiz 7 **27 October, 2022**

Name: SOLUTIONS / 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [3 points] Find a formula for the nth term a_n of the following recursively defined sequence:

$$a_1 = 1 \text{ and } a_{n+1} = (n+1)a_n \text{ for } n \geq 1$$

$a_2 = (1+1)a_1 = 2a_1 = 2\cdot 1$

$a_3 = (2+1)a_2 = 3\cdot 2\cdot 1$

$a_4 = (3+1)a_3 = 4\cdot 3\cdot 2\cdot 1$

so $a_n = n!$

2. [4 points] Determine the limit of the sequence or explain why the sequence diverges:

$$a_n = \ln\left(\frac{n+2}{n^2-3}\right)$$

"$\frac{1}{\infty} = 0$"

$$\lim_{n\to\infty} \frac{n+2}{n^2-3} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{n\to\infty} \frac{1}{2n} = 0$$

and $\frac{n+2}{n^2-3}$ are positive values for large n

So

$$\lim_{n\to\infty} \ln\left(\frac{n+2}{n^2-3}\right) = \lim_{x\to 0^+} \ln(x) = \boxed{-\infty}$$

(diverges)

3. [4 points] Compute and simplify the fourth partial sum S_4 of the series $\sum_{n=1}^{\infty} a_n$ which has nth term $a_n = \frac{1}{n}$.

$$S_4 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{12+6+4+3}{12}$$

$$= \frac{25}{12}$$

4. [4 points] Use sigma notation to write a simplified expression for the infinite series.

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n}}{n+1}$$

$$= \sum_{n=0}^{\infty} \frac{\cos(\pi n)}{n+1}$$

any of these are correct

5. [10 points] State whether the given series converges or diverges, and explain why. If the series converges, find its sum.

a. $1+\frac{e}{\pi}+\frac{e^2}{\pi^2}+\frac{e^3}{\pi^3}+\ldots$

geometric with

$a=1, \quad r=\frac{e}{\pi}$

since $|r|<1$, converges

$S=\frac{a}{1-r}$

$$S=\sum_{n=0}^{\infty}\left(\frac{e}{\pi}\right)^n=\frac{1}{1-\frac{e}{\pi}}=\frac{\pi}{\pi-e}$$

b. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$

partial fractions:

$$\frac{1}{n(n+1)}=\frac{A}{n}+\frac{B}{n+1}$$

$$1=A(n+1)+Bn$$

$$=(A+B)n+A$$

$$\left.\begin{aligned}A+B&=0\\ A&=1\end{aligned}\right\} B=-1$$

$$S_k=\sum_{n=1}^{k}\frac{1}{n(n+1)}$$

$$=\sum_{n=1}^{k}\frac{1}{n}-\frac{1}{n+1}$$

$$=\frac{1}{1}-\cancel{\frac{1}{2}}+\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}+\cancel{\ldots}+\cancel{\frac{1}{k}}-\frac{1}{k+1}=1-\frac{1}{k+1}$$

So:

$$S=\sum_{n=1}^{\infty}\frac{1}{n(n+1)}=\lim_{k\to\infty}S_k=\lim_{k\to\infty}1-\frac{1}{k+1}=1-0$$

$$=1$$

converges

EC. **[1 points]** (**Extra Credit**) Write the repeating decimal $x = 2.787878\cdots = 2.\overline{78}$ as a rational number. That is, find integers N and M so that $x = N/M$.

subtract

$x = 2.7878\cdots$

$100x = 278.7878\cdots$

$99x = 278 - 2 = 276$

$x = \frac{276}{99} = \frac{92}{33}$

either

$$\begin{array}{r} 92 \\ 3\overline{)276} \\ \underline{27} \\ 6 \\ \underline{6} \\ 0 \end{array}$$

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