Name: ______________________________ Quiz 10

Math F252X-901, Calculus II Fall 2024

Thirty minutes maximum. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably simplified form.

1. Determine whether the following series converge or diverge. Justify your answers.

   (a) (8 points.) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{\pi}+\frac{1}{n}\right)^{n}$

   $$\lim_{n\to\infty}\sqrt[n]{\left(\frac{1}{\pi}+\frac{1}{n}\right)^{n}}=\lim_{n\to\infty}\frac{1}{\pi}+\frac{1}{n}$$
   $$=\frac{1}{\pi}<1$$

   So by the root test, the series converges.

   (b) (8 points.) $\displaystyle\sum_{n=1}^{\infty}\frac{10^{n}}{n!}$

   $$\lim_{n\to\infty}\frac{10^{n+1}}{(n+1)!}\cdot\frac{n!}{10^{n}}=\lim_{n\to\infty}\frac{10}{n+1}=0<1$$

   So by the ratio test, the series converges.

(c) (8 points.) $\displaystyle\sum_{k=1}^{\infty} \frac{(2k)!}{k^{2k}}$ hint: be VERY careful with grouping symbols!

$$\lim_{n\to\infty} \frac{(2(k+1))!}{(k+1)^{2(k+1)}} \cdot \frac{k^{2k}}{(2k)!} = \lim_{n\to\infty} \frac{k^{2k}(2k+2)!}{(k+1)^{2k+2}(2k)!}$$

$$= \lim_{n\to\infty} \frac{(2k+2)(2k+1)k^{2k}}{(k+1)^{2k+2}}$$

$$= \lim_{n\to\infty} \frac{(2k)(2k)(k^{2k})}{(k+1)^{2k+2}} \text{ (why can we ignore "+1" and "+2"?)}$$

$$= \lim_{n\to\infty} \frac{4k^{2k+2}}{(k+1)^{2k+2}}$$

$$= \lim_{n\to\infty} \frac{4}{\left(1+\frac{1}{k}\right)^{2k+2}}$$

$$= \frac{4}{e^2} < 1$$

So the series converges by the ratio test.

(d) (8 points.) $\displaystyle\sum_{n=0}^{\infty} \frac{n^e}{e^n}$

$$\lim_{n\to\infty} \sqrt[n]{\frac{n^e}{e^n}} = \frac{\sqrt[n]{n}^e}{e} = \frac{1}{e} < 1$$

So the series converges by the root test.

2. Consider the power series $\displaystyle\sum_{k=0}^{\infty} \frac{3k+1}{k!}(x+4)^k$.

(a) (4 points.) State the center of the power series: -4

(b) (4 points.) State one value of x for which the power series converges: $x = -4$