Math 252: Quiz 1 **Fall 2025**

Name: Solutions

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24 points possible; each part is worth 2 points. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

1. [12 points] Compute the derivatives of the following functions.

a. $f(\theta) = \arcsin(\theta) - 2^\theta + \frac{1}{e}$ ← a fixed constant

Formulas we told you you needed to know!

$$f'(\theta) = \frac{1}{\sqrt{1-\theta^2}} - (\ln 2)\, 2^\theta$$

b. $f(x) = \cos(ax^2 + b)$ for fixed constants b and c

Like practice quiz 1 # c.

$$f'(x) = -\sin(ax^2+b)\,(2ax)$$

$$= -2ax \sin(ax^2+b)$$

c. $H(t) = t^{2.4} \ln(2t+1)$

product rule + chain rule w/ logs.

$$H'(t) = 2.4\,t^{1.4} \ln(2t+1) + t^{2.4} \cdot \left(\frac{2}{2t+1}\right)$$

$(\sin x)^2$. chain rule!

d. $f(x) = 5e^{x/2} + \sin^2(x)$

two simple chain rules.

$$f'(x) = 5 \cdot \left(\tfrac{1}{2}\right) \cdot e^{\frac{x}{2}} + 2\sin(x)\cos(x)$$

e. $h(x) = \dfrac{x + \sin(x)}{x + \pi}$

not required!

quotient rule

$$h'(x) = \frac{(1+\cos(x))(x+\pi) - 1(x+\sin(x))}{(x+\pi)^2}$$

f. $h(x) = \dfrac{1}{5x} + \dfrac{\sqrt{x}}{10} = \dfrac{1}{5}x^{-1} + \dfrac{1}{10}x^{\frac{1}{2}}$

Know how to rewrite exponents

$$h'(x) = -\frac{1}{5}x^{-2} + \frac{1}{10} \cdot \frac{1}{2} \cdot x^{-1/2}$$

$$= \frac{-1}{5x^2} + \frac{1}{20\sqrt{x}}$$

2. [12 points] Compute the following antiderivatives (indefinite integrals) and definite integrals. Remember that antiderivatives need a "$+C$".

basic rules

a. $\int \sin(2x) + \sqrt{x}\, dx \quad = \int \left(\sin(2x) + x^{\frac{1}{2}}\right) dx$

$= -\frac{1}{2}\cos(2x) + \frac{2}{3}x^{3/2} + C$

b. $\int (6x-5)^{(1/4)}\, dx = \frac{1}{6}\int u^{\frac{1}{4}}\, du \longleftarrow \begin{cases} u = 6x-5 \\ du = 6\, dx \\ \frac{1}{6}du = dx \end{cases}$

$= \frac{1}{6}\cdot\frac{4}{5}u^{\frac{5}{4}} + C$

Like HW §1.5 # 274

$= \frac{2}{15}(6x-5)^{5/4} + C$

c. $\int \frac{\sec^2(x)}{\tan(x)}\, dx = \int \frac{du}{u} \longleftarrow \begin{cases} u = \tan x \\ du = \sec^2 x\, dx \end{cases}$

$= \ln|u| + C = \ln|\tan x| + C$

Like practice quiz 1 # f

d. $\displaystyle\int \frac{2x^3+x^2-12}{x^2}\,dx = \int \left(2x + 1 - 12x^{-2}\right) dx$

Simplify first.

$= x^2 + x + 12x^{-1} + C$

e. $\displaystyle\int_{-1}^{0} \frac{x}{(2+x)^2}\,dx = \int_1^2 \frac{u-2}{u^2}\,du$

$\begin{cases} u = 2+x \\ du = dx \\ u-2 = x \end{cases}$ | If $x=-1$, $u=1$; $x=0$, $u=2$

$\displaystyle\int_1^2 \left(u^{-1} - 2u^{-2}\right) du = \ln u - \frac{2}{3}u^{3}\Big|_1^2$

Like HW 1.5 #294

$= \left(\ln 1 - \frac{2}{3}\right) - \left(\ln 2 - \frac{16}{3}\right) = -\ln(2) + \frac{14}{3}$

f. $\displaystyle\int \frac{e^t}{1+e^{(2t)}}\,dt = \int \frac{du}{1+u^2}$ $\leftarrow$ $\begin{cases} \text{let } u = e^t \\ du = e^t \end{cases}$

$= \arctan(u) + C$

$= \arctan(e^t) + C$

HW § 1.7 #424 exactly