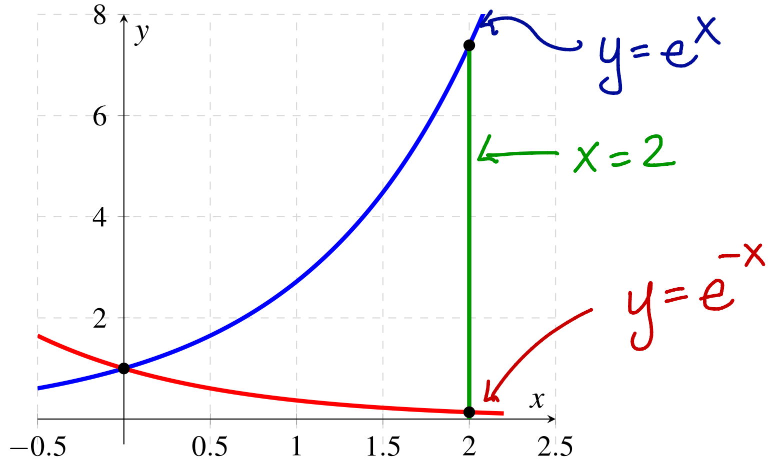
Math 252: Quiz 2 **4 Sept 2025**

Name: Solutions ____________ / 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. (9 points) The region R, bounded by $y = e^x$, $y = e^{-x}$ and $x = 2$ is sketched below.

like §2.1 #2, 4, 6, 8, 10



(a) Label each of the three graphs with the appropriate equation.

(b) Set up and evaluate an integral to calculate the area of region R. Your answer should be in reasonable simplified form.

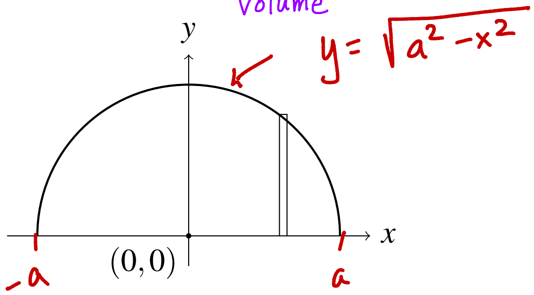
$$A = \int_0^2 (e^x - e^{-x})\,dx = e^x + e^{-x}\Big]_0^2 = (e^2 + e^{-2}) - (e^0 + e^{-0})$$

$$= e^2 + \frac{1}{e^2} - 2$$

Due to the typo, there are two solutions below. One solution for the $\boxed{\text{volume}}$ (intended problem) and one for $\boxed{\text{area}}$.

§2.2 #68 exactly

2. (5 points) Suppose a solid has a base that is the top-half of a circle of radius a and slices perpendicular to the base are squares. The base and a sample rectangle are below. Set up an intergral to find its ~~area.~~ volume You do not need to evaluate the integral.



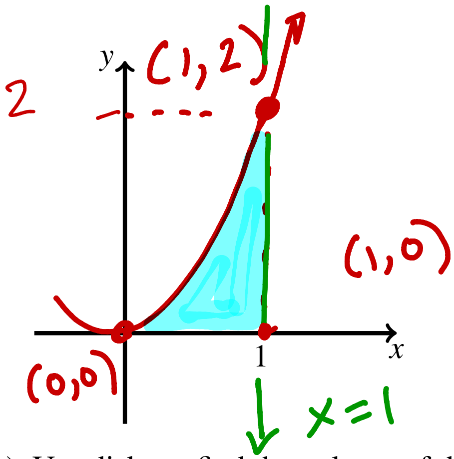
$$V = 2\int_0^a \left(\sqrt{a^2 - x^2}\right)^2 dx = 2\int_0^a (a^2 - x^2)\,dx$$

$$A = 2\int_0^a \sqrt{a^2 - x^2}\,dx$$

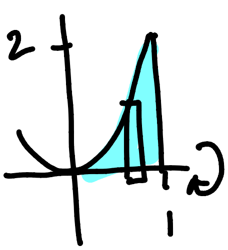
like §2.2 #76–98

3. (11 points) Let R be the region bounded by $y = 2x^2$, $y = 0$, and $x = 1$.

(a) Sketch the region R, label the graphs with their equations, and label the points of intersection with their coordinates.



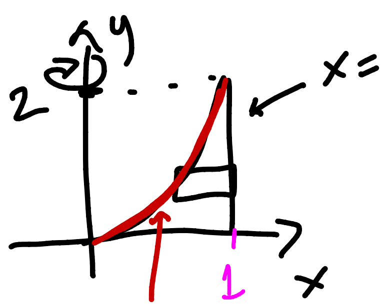
(b) Use disks to find the volume of the solid obtained by rotating R about the x-axis.



$$V=\int_0^1 \pi\left(2x^2\right)^2 dx = 4\pi\int_0^1 x^4\,dx = 4\pi\cdot\frac{1}{5}x^5\Big]_0^1$$

$$=\frac{4\pi}{5}$$

(c) Use washers to find the volume of the solid obtained by rotating R about the y-axis.



$$V=\pi\int_0^2\left[(1)^2-\left(\sqrt{\frac{y}{2}}\right)^2\right]dy$$

$$=\pi\int_0^2\left(1-\frac{y}{2}\right)dy=\pi\left(y-\frac{1}{4}y^2\right)\Big|_0^2$$

$$=\pi\left(2-\frac{1}{4}(2)^2\right)=\pi(2-1)=\pi$$

$y=2x^2$

$x=\sqrt{\frac{y}{2}}$