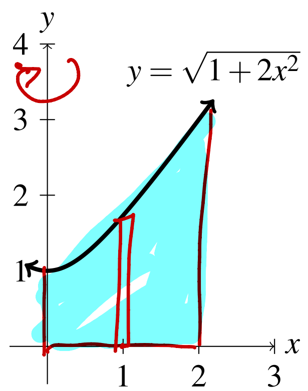
Name: Solutions ______ / **25**

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. (9 points) The region R is bounded by $y=\sqrt{1+2x^2}$, $x=2$, and the x- and y- axes. Sketch the region R on the graph below and answer the questions.



Shells

$$V = 2\pi \int_a^b x f(x)\, dx$$

← mala bold.

(a) Use shells to find the volume of the solid obtained by rotating the region R about the $\boxed{y\text{-axis}}$. (You must **set-up** an integral and **evaluate** it.)

$$V = 2\pi \int_0^2 x(1+2x^2)^{1/2}\, dx$$

let $u = 1+2x^2$
$du = 4x\,dx$
$\frac{1}{4}du = x\,dx$
If $x=0$, $u=1$
If $x=2$, $u=9$

$$= 2\pi \cdot \frac{1}{4}\int_1^9 u^{1/2}\, du$$

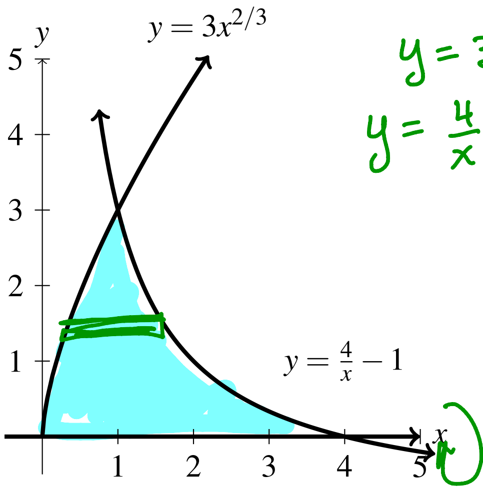
$$= \frac{\pi}{2}\cdot\frac{2}{3}u^{3/2}\Big|_1^9 = \frac{\pi}{3}\left(9^{3/2} - 1^{3/2}\right) = \frac{\pi}{3}(27-1) = \frac{26\pi}{3}$$

(b) Give at least one reason why the method of cylindrical shells might be better than disks or washers for the problem in part (a).

Disks/washers would require two integrals and solving $y=\sqrt{1+2x^2}$ for x.

space

2. (5 points) Suppose the region R is bounded by $y = 3x^{2/3}$, $y = \frac{4}{x} - 1$ and the x-axis. (Graphed below.) **Set up but do not evaluate** an integral for finding the volume of the solid generated by rotating R about the $\boxed{x\text{-axis}}$. **Use shells.**



Solve equations for x:

$y = 3x^{2/3}$ becomes $x = \left(\frac{y}{3}\right)^{3/2}$

$y = \frac{4}{x} - 1$ becomes $\frac{y+1}{4} = \frac{1}{x}$ or $x = \frac{4}{y+1}$

$$V = 2\pi \int_0^3 y\left(\frac{4}{y+1} - \left(\frac{y}{3}\right)^{3/2}\right) dy$$

Formulas: arc length $= \int_a^b \sqrt{1 + (f'(x))^2}\, dx$ surface area $= \int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\, dx$

3. (3 points) **Set up but do not evaluate** an integral to calculate the length of the function $y = \ln(x^2 + 1)$ from $x = 0$ to $x = 10$.

$y' = \frac{2x}{x^2+1}$; $L = \int_0^{10} \sqrt{1 + \frac{4x^2}{(x^2+1)^2}}\, dx$

4. (8 points) Find the surface area of the volume generated when the curve $y = \sqrt{4 - x^2}$ from $x=0$ to $x=1$ revolves around the x-axis. (You can evaluate this integral. The points will be distributed as: 5 points to correctly set up the integral and 3 points for the work to correctly evaluate it.)

$f(x) = (4 - x^2)^{1/2}$

$f'(x) = \frac{1}{2}(4 - x^2)^{-1/2}(-2x)$

$= \frac{-x}{\sqrt{4 - x^2}}$

$(f'(x))^2 = \frac{x^2}{4 - x^2}$

$$A = 2\pi \int_0^1 \left(\sqrt{4 - x^2}\sqrt{1 + \frac{x^2}{4 - x^2}}\right) dx$$

$$= 2\pi \int_0^1 \sqrt{(4 - x^2) + x^2}\, dx$$

$$= 2\pi \int_0^1 \sqrt{4}\, dx = 4\pi$$