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Math 252: Quiz 4 **18 Sept 2025**

Name: Solutions ______ / 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. (4 points) A 1-dimensional metal rod is 5 meters long (starting at $x = 0$) and has density function $\rho(x) = e^{2x}$ kg/m.

   (a) (1 point) Compare the density of the rod at $x = 1$ and at $x = 4$. Where is the rod more dense? Explain your reasoning.

   $e(1) = e$

   $e(4) = e^8$

   Since $e(4) > e(1)$, the rod is more dense at 4 compared to at 1.

   (b) (3 points) Find the mass of the 1-dimensional rod. Give units with your answer.

   $$m = \int_0^5 e^{2x}\,dx = \frac{1}{2}e^{2x}\Big|_0^5 = \frac{1}{2}\left(e^{10}-1\right) \text{ kg}$$

2. (8 points) A spring has a natural length of 2 meters. It requires 5 J of work to stretch the spring to a length of 2.5 meters.

   (a) (3 points) Find the spring constant k in Hooke's Law. Hooke's Law $F = kx$

   $$\underset{W}{5\,J} = \int_0^{\frac{1}{2}} \underbrace{kx}_{F}\cdot\underbrace{dx}_{d} = \frac{k}{2}\cdot(x)^2\Big|_0^{1/2} = \frac{k}{8}.$$ So $k = 40$ (N/m)

   (b) (2 points) Use your answer from part (a) to determine how much force the spring exerts if the spring is displaced exactly 0.5 meters from its natural position. Include units with your answer.

   $F = 40x = 40 \cdot \frac{1}{2} = 20$ N.

   (c) (3 points) How much work would it take to stretch the spring from 3 meters to 4 meters? Include units with your answer.

   displacement is from $x=1$ to $x=2$

   $$W = \int_1^2 40x\,dx = 20x^2\Big|_1^2 = 20(16-1) = 20\cdot 15 = 60 \text{ J}$$

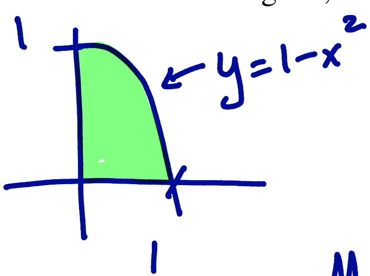
3. (4 points) Calculate the center of mass for the collection of point masses given where x_i gives the location of the point mass on the x-axis and m_i is the mass.

location	mass
$x_1 = 1$	$m_1 = 6$
$x_2 = 3$	$m_2 = 2$
$x_3 = 4$	$m_3 = 1$

$$\bar{x} = \frac{M}{m} = \frac{1\cdot 6 + 3\cdot 2 + 4\cdot 1}{6+2+1} = \frac{6+6+4}{9} = \frac{16}{9}$$

4. (9 points) Let R be the region in the first quadrant bounded by $f(x) = 1 - x^2$. (Making a rough sketch of R would probably be helpful here.)

   (a) (5 points) Find the $\bar{x}$, the x-coordinate of the centroid of R. (This will require evaluating two integrals, one to calculate m and another for M_y.)



$$m = \int_0^1 (1 - x^2)\,dx = x - \tfrac{1}{3}x^3 \Big|_0^1 = 1 - \tfrac{1}{3} = \boxed{\tfrac{2}{3}}$$

$$M_y = \int_0^1 x(1-x^2)\,dx = \int_0^1 (x - x^3)\,dx = \tfrac{1}{2}x^2 - \tfrac{1}{4}x^4 \Big|_0^1$$

$$= \tfrac{1}{2} - \tfrac{1}{4} = \boxed{\tfrac{1}{4}}$$

$$\bar{x} = \frac{1/4}{2/3} = \tfrac{1}{4}\cdot\tfrac{3}{2} = \boxed{3/8}$$

answer

   (b) (2 points) **Set up but do not evaluate** the third integral you would need to determine $\bar{y}$, the y-coordinate of the centroid of R.

$$M_x = \tfrac{1}{2}\int_0^1 (1 - x^2)^2\,dx \quad \left(= \tfrac{1}{2}\int_0^1 (1 - 2x^2 + x^4)\,dx = \tfrac{1}{2}\left(x - \tfrac{2}{3}x^3 + \tfrac{1}{5}x^5\right)\Big|_0^1 \right.$$

$$\left. = \tfrac{1}{2}\left(1 - \tfrac{2}{3} + \tfrac{1}{5}\right) = \tfrac{1}{2}\left(\tfrac{15}{15} - \tfrac{10}{15} + \tfrac{3}{15}\right) = \tfrac{4}{15}; \text{ so } \bar{y} = \frac{4/15}{2/3} = \tfrac{4}{15}\cdot\tfrac{3}{2} = \tfrac{2}{5} \right)$$

The $\boxed{\text{Red}}$ is strictly FYI.

   (c) (2 points) Suppose someone calculated $\bar{y} = 2/5$. Is this value reasonable/plausible? Justify your conclusion.

Yes. It's clear that $\bar{y}$ should be in the bottom half of the figure. Since $\frac{2}{5} < \frac{1}{2}$, $\bar{y}$ is plausible.