Math 252: Quiz 4 **18 Sept 2025**

Name: _____ / **25**

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. (4 points) A 1-dimensional metal rod is 5 meters long (starting at $x = 0$) and has density function $\rho(x) = e^{2x}$ kg/m.

   (a) (1 point) Compare the density of the rod at $x = 1$ and at $x = 4$. Where is the rod more dense? Explain your reasoning.

   (b) (3 points) Find the mass of the 1-dimensional rod. Give units with your answer.

2. (8 points) A spring has a natural length of 2 meters. It requires 5 J of **work** to stretch the spring to a length of 2.5 meters.

   (a) (3 points) Find the spring constant k in Hooke's Law.

   (b) (2 points) Use your answer from part (a) to determine how much **force** the spring exerts if the spring is displaced exactly 0.5 meters from its natural position. Include units with your answer.

   (c) (3 points) How much **work** would it take to stretch the spring from 3 meters to 4 meters? Include units with your answer.

3. (4 points) Calculate the center of mass for the collection of point masses given where x_i gives the location of the point mass on the x-axis and m_i is the mass.

location	mass
$x_1 = 1$	$m_1 = 6$
$x_2 = 3$	$m_2 = 2$
$x_3 = 4$	$m_3 = 1$

4. (9 points) Let R be the region in the first quadrant bounded by $f(x) = 1 - x^2$. (Making a rough sketch of R would probably be helpful here.)

   (a) (5 points) Find the $\bar{x}$, the x-coordinate of the **centroid** of R. Recall that the **centroid** is the same as finding the **center of mass** when the density, ρ, is 1. This will require evaluating two integrals, one to calculate m and another for M_y.

   (b) (2 points) **Set up but do not evaluate** the third integral you would need to determine $\bar{y}$, the y-coordinate of the centroid of R.

   (c) (2 points) Suppose someone calculated $\bar{y} = 2/5$. Is this value reasonable/plausible? Justify your conclusion.