Math 252: Quiz 5 **25 Sept 2025**

Name: Solutions ______ / 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. (12 points) Evaluate the integrals using Integration by Parts.

(a) $\int_1^2 xe^x\,dx$ $= xe^x\Big|_1^2 - \int_1^2 e^x\,dx = (2e^2 - e) - \left[e^x\right]_1^2$

$u = x \quad dv = e^x\,dx$

$du = dx \quad v = e^x$

$= 2e^2 - e - (e^2 - e^1) = \boxed{e^2}$

(b) $\int \arcsin(x)\,dx$ $= x\arcsin(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$\boxed{\begin{array}{ll} u = \arcsin x & dv = dx \\ du = \frac{1}{\sqrt{1-x^2}}\,dx & v = x \end{array}}$

$= x\arcsin x - \int (1-x^2)^{-1/2}\, x\,dx$

u-sub
$u = 1-x^2$
$du = -2x\,dx$
$-\frac{1}{2}du = x\,dx$

$= x\arcsin x + \frac{1}{2}\int u^{-1/2}\,du$

$= x\arcsin x + u^{1/2} + C$

$\boxed{= x\arcsin x + \sqrt{1-x^2} + C}$

(c) $\int x^2\ln(x^4)\,dx$ $= 4\int x^2\ln(x)\,dx$

choose $u = \ln(x) \quad dv = x^2$

$du = \frac{1}{x}dx \quad v = \frac{1}{3}x^3$

$= 4\left(\frac{1}{3}x^3\ln(x) - \frac{1}{3}\int x^3 \cdot \frac{1}{x}\,dx\right)$

$= \frac{4}{3}\left(x^3\ln(x) - \int x^2\,dx\right)$

$\boxed{= \frac{4}{3}\left(x^3\ln(x) - \frac{1}{3}x^3\right) + C}$ $= \frac{x^3}{3}\left(\ln(x^4) - \frac{1}{3}\right) + C$

2. (13 points) Evaluate the integrals below. Integration by Parts is not needed. (The first problem is 4 points. The others are 3 points.)

(a) $\displaystyle\int_e^{e^2} \frac{\ln(\ln(x))}{x\ln(x)}\,dx = \int_0^{\ln(2)} u\,du = \frac{1}{2}u^2\Big|_0^{\ln(2)}$

Let $u = \ln(\ln(x))$

$du = \frac{1}{\ln x}\cdot\frac{1}{x}\cdot dx$

$x = e,\ u = \ln(\ln e) = \ln(1) = 0$

$x = e^2,\ u = \ln(\ln(e^2)) = \ln(2)$

$$\boxed{= \frac{1}{2}\left(\ln(2)\right)^2}$$

(b) $\displaystyle\int \tan(x)\,dx = \int \frac{\sin x}{\cos x}\,dx = \boxed{-\ln|\cos x| + C}$

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a better way!

choose $u = \sec x$

$du = \sec x\tan x\,dx$

$\int \sec^3 x\,(\sec x\tan x\,dx) = \int u^3\,du$

$= \frac{1}{4}u^4 + C = \frac{1}{4}(\sec x)^4 + C$

(c) $\displaystyle\int \tan(x)\sec^4(x)\,dx = \int \tan x\,\sec^2 x\,\sec^2 x\,dx$

$= \int \tan x(\tan^2 x + 1)\sec^2 x\,dx$

Let $u = \tan x$

$du = \sec^2 x\,dx$

$= \int u(u^2+1)\,du = \int(u^3+u)\,dx = \frac{1}{4}u^4 + \frac{1}{2}u^2 + C$

$$\boxed{= \frac{1}{4}(\tan x)^4 + \frac{1}{2}(\tan x)^2 + C}$$

(d) $\displaystyle\int \sin^3(x)\,dx = \int \sin^2 x\cdot\sin x\,dx = \int(1-\cos^2 x)\sin x\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$= -\int(1-u^2)\,du$

$= \int(u^2-1)\,du = \frac{1}{3}u^3 - u + C = \boxed{\frac{1}{3}\cos^3 x - \cos x + C}$