Math 252: Quiz 6 **16 October 2025**

Name: Solutions

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30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. (2 points) Find the first three terms of the sequence defined below, starting with $n = 1$.

$$a_1 = 3 \text{ and } a_n = 2a_{n-1} - 1 \text{ for } n \geq 2$$

$a_1 = 3$
$a_2 = 2 \cdot a_1 - 1 = 6 - 1 = 5$
$a_3 = 2 \cdot a_2 - 1 = 10 - 1 = 9$

$$\boxed{\begin{array}{l} a_1 = 3 \\ a_2 = 5 \\ a_3 = 9 \end{array}}$$

2. (8 points) Determine the limit of the sequence or show that the sequence diverges. If it converges, find its limit. Answers with no work will earn no credit.

(a) $a_n = \frac{\sqrt{n}}{\sqrt{3n+1}}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{3n+1}} = \lim_{n\to\infty} \sqrt{\frac{n}{3n+1} \cdot \frac{1/n}{1/n}} = \lim_{n\to\infty} \sqrt{\frac{1}{3+\frac{1}{n}}}$$

$$= \sqrt{\frac{1}{3+0}} = \frac{1}{\sqrt{3}}.$$

The sequence converges to $\frac{1}{\sqrt{3}}$.

(b) $a_n = \frac{n}{2^n}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = \lim_{x\to\infty} \frac{x}{2^x} \overset{H}{=} \lim_{x\to\infty} \frac{1}{(\ln 2) 2^x} = 0$$

form $\frac{\infty}{\infty}$

Convert to a continuous fcn of x. So L'Hopital's rule applies

$\boxed{\text{The sequence converges to zero}}$

3. (3 points) Write a general formula for the sequence $\left\{ \frac{1}{3}, \frac{-1}{6}, \frac{1}{9}, \frac{-1}{12}, \frac{1}{15}, \frac{-1}{18}, \cdots \right\}$. A complete answer includes **both** a formula for a_n and clear indication of where the index n starts. (Does n start a zero? one? two?)

$$a_n = \frac{(-1)^{n-1}}{3n} \quad \text{for } n \geq 1$$

4. (10 points) Evaluate the following improper integrals, if possible. If the integral is not convergent, answer divergent. **You must use correct notation and show your work.** Answers without work will earn no credit.

   (a) $\displaystyle\int_1^4 \frac{1}{\sqrt{12-3x}}\,dx = \lim_{t\to 4^-} \int_1^t (12-3x)^{-1/2}\,dx = \lim_{t\to 4^-} -\frac{2}{3}(12-3x)^{1/2}\Big]_1^t$

   $\displaystyle= \lim_{t\to 4^-} -\frac{2}{3}\left(\sqrt{12-3t}-\sqrt{9}\right) = -\frac{2}{3}(0-3) = 2$

   (b) $\displaystyle\int_3^\infty \frac{6}{1+x^2}\,dx = \lim_{t\to\infty} \int_3^t \frac{6}{1+x^2}\,dx = \lim_{t\to\infty} 6\arctan(x)\Big]_3^t$

   $\displaystyle= \lim_{t\to\infty} 6\left(\arctan(t)-\arctan(3)\right) = 6\left(\frac{\pi}{2}-\arctan(3)\right)$

   $= 3\pi - \arctan(3)$

5. (2 points) **Without integrating** determine whether the integral $\displaystyle\int_3^\infty \frac{6}{10x+x^2}\,dx$ converges or diverges. Justify your conclusion. An answer without any work will earn no credit. (HINT: Use your work in Problem 1, above.)

<u>Ans</u>: It converges.

<u>Justification</u>: $0 \le \dfrac{6}{10x+x^2} < \dfrac{6}{1+x^2}$ for $x \geq 3$ and $\displaystyle\int_3^\infty \frac{6}{1+x^2}\,dx$ converges. So $\displaystyle\int_3^\infty \frac{6}{10x+x^2}\,dx$ must also converse.