Math 252: Quiz 6 **16 October 2025**

Name: Solutions

________ / **25**

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. (10 points) Evaluate the following improper integrals, if possible. If the integral is not convergent, answer divergent. **You must use correct notation and show your work.** Answers without work will earn no credit.

   (a) $\int_2^\infty \frac{10}{1+x^2}\,dx = \lim_{t\to\infty} \int_2^t \frac{10}{1+x^2}\,dx = \lim_{t\to\infty} 10\arctan x \Big|_2^t$

   $= \lim_{t\to\infty} \left(10\arctan(t) - 10\arctan(2)\right) = \frac{10\cdot\pi}{2} - 10\arctan(2)$

   $= \boxed{5\pi - 10\arctan(2)}$

   (b) $\int_1^5 \frac{1}{\sqrt{15-3x}}\,dx = \lim_{t\to 5^-} \int_1^t (15-3x)^{-1/2}\,dx = \lim_{t\to 5^-} -\frac{2}{3}(15-3x)^{\frac{1}{2}}\Big]_1^t$

   $= \lim_{t\to 5^-} -\frac{2}{3}\left(\sqrt{15-3t} - \sqrt{12}\right) = -\frac{2}{3}\left(0 - \sqrt{12}\right) = \frac{2\sqrt{12}}{3} = \boxed{\frac{4\sqrt{3}}{3}}$

2. (2 points) **Without integrating** determine whether the integral $\int_2^\infty \frac{10}{10x+x^2}\,dx$ converges or diverges. Justify your conclusion. An answer without any work will earn no credit. (HINT: Use your work in Problem 1, above.)

   Ans: The integral converges.

   Justification: $0 \le \frac{10}{10x+x^2} < \frac{10}{1+x^2}$ and $\int_2^\infty \frac{10}{1+x^2}\,dx$ converges. So $\int_2^\infty \frac{10}{10x+x^2}\,dx$ is forced to converge.

v1

3. (2 points) Find the first three terms of the sequence defined below, starting with $n = 1$.

$$a_1 = 4 \text{ and } a_n = 3a_{n-1} - 2 \text{ for } n \geq 2$$

$a_1 = 4,$
$a_2 = 3a_1 - 2 = 3 \cdot 4 - 2 = 12 - 2 = 10,$
$a_3 = 3 \cdot a_2 - 2 = 3 \cdot 10 - 2 = 28$

$$\boxed{\begin{array}{l} a_1 = 4 \\ a_2 = 10 \\ a_3 = 28 \end{array}}$$

4. (3 points) Write a general formula for the sequence $\{\frac{1}{2}, \frac{-1}{4}, \frac{1}{6}, \frac{-1}{8}, \frac{1}{10}, \frac{-1}{12}, \cdots\}$. A complete answer includes **both** a formula for a_n and clear indication of where the index n starts. (Does n start a zero? one? two?)

$$a_n = \frac{(-1)^{n+1}}{2n} \quad \text{for } n \geq 1$$

convert to a continuous function of x.

5. (8 points) Determine the limit of the sequence or show that the sequence diverges. If it converges, find its limit. Answers with no work will earn no credit.

(a) $a_n = \frac{n}{3^n}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{3^n} = \lim_{x\to\infty} \frac{x}{3^x} \overset{H}{=} \lim_{x\to\infty} \frac{1}{(\ln 3)3^x} = 0$$

$\boxed{\text{The sequence converges to zero.}}$

(b) $a_n = \frac{\sqrt{n}}{\sqrt{2n+1}}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{2n+1}} = \lim_{n\to\infty} \sqrt{\frac{n}{2n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}} = \lim_{n\to\infty} \sqrt{\frac{1}{2+\frac{1}{n}}}$$

$$= \sqrt{\frac{1}{2+0}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}.$$

$\boxed{\text{The sequence converges to } \frac{1}{\sqrt{2}}}$