Math 252: Quiz 9 **6 November 2025**

Name: Solutions.

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30 minutes maximum. No aids (book, calculator, etc.) are permitted. **Show all work** and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. (10 points) Use either the ratio test or the root test as appropriate to determine whether the series converges or diverges or state that the test is inconclusive. State the test that you are using.

(a) $\sum_{n=1}^{\infty} \frac{3^{2n}(n!)^2}{(2n)!}$ ratio test

$$\lim_{n\to\infty} \frac{3^{2n+2}((n+1)!)^2}{(2n+2)!} \cdot \frac{(2n)!}{3^{2n}(n!)^2} = \lim_{n\to\infty} \frac{3^2(n+1)^2}{(2n+2)(2n+1)} = \lim_{n\to\infty} \frac{9(n^2+2n+1)}{4n^2+6n+1} = \frac{9}{4} > 1$$

Thus, the series diverges.

(b) $\sum_{n=1}^{\infty} \left(\frac{e+n}{en}\right)^n$ root test

$$\lim_{n\to\infty} \frac{e+n}{en} = \frac{1}{e} < 1$$ So the series converges.

2. (4 points) For which p-values does the series $\sum_{n=1}^{\infty} \frac{2^n}{n^p}$ converge? Justify your conclusion.

It never converges. So no values of p will give convergence.

Justification: Ratio test

$$\lim_{n\to\infty} \frac{2^{n+1}}{(n+1)^p} \cdot \frac{n^p}{2^n} = 2 \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^p = 2 \cdot 1 = 2 > 1$$

3. (4 points) Suppose the series $\sum_{n=0}^{\infty} a_n(x-5)^n$ converges at $x=4$. Can you conclude that the series converges at $x=4.7$? Justify your conclusion.

Yes. The series does converge at $x=4.7$

Justification: The series must converge on an interval centered at $x=5$. Since we are told it converges at $x=4$, we know the interval of convergence must contain the interval $[4,6)$.

4. (7 points) Consider the series $\sum_{n=1}^{\infty} \frac{x^n}{n^{1/3}}$.

(a) Find R, the radius of convergence of the series.

Ratio Test

$$\lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)^{1/3}}\cdot\frac{n^{1/3}}{x^n}\right| = |x|\lim_{n\to\infty}\sqrt[3]{\frac{n}{n+1}} = |x| < 1$$

So $R=\underline{1}$

(b) Determine the interval of convergence of the series. (Make sure to check any endpoints, if they exist.)

Check $x=1$: $\sum_{n=1}^{\infty}\frac{1}{n^{1/3}}$. This is a divergent p-series.

Check $x=-1$: $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^{1/3}}$. This converges by A.S.T. (see below)

AST.: ① $\lim_{n\to\infty}\frac{1}{\sqrt[3]{n}}=0$ ✓, ② $b_{n+1}=\frac{1}{\sqrt[3]{n+1}} < \frac{1}{\sqrt[3]{n}}=b_n$ ✓

<u>ANSWER</u>: I.O.C. $[-1,1)$