

Name: _____

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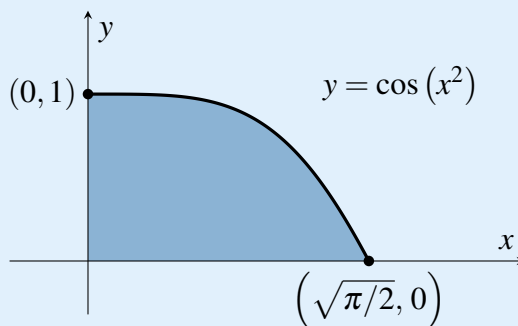
30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. Consider the region in the first quadrant bounded by the curves $y = \cos(x^2)$, $y = 0$, $x = 0$.

- (a) (4 points) Sketch the region. **Clearly label any important points.**

Solution

On $x \geq 0$ the curve $y = \cos(x^2)$ first meets $y = 0$ when $x^2 = \frac{\pi}{2}$, so $x = \sqrt{\pi/2}$.



- (b) (9 points) Find the volume of the solid obtained by rotating the region about the y-axis.

Solution

Rotating about the y-axis, a cylindrical shell at x has radius x and height $\cos(x^2)$:

$$V = \int_0^{\sqrt{\pi/2}} 2\pi x \cos(x^2) dx.$$

Substituting $u = x^2$, $du = 2x dx$, with $u = 0$ when $x = 0$ and $u = \frac{\pi}{2}$ when $x = \sqrt{\pi/2}$,

$$V = \pi \int_0^{\pi/2} \cos u du = \pi \left[\sin u \right]_0^{\pi/2} = \pi(1 - 0).$$

$$V = \pi$$

2. (6 points) Set up, but **do not evaluate** an integral that computes the arc length of the curve $y = x^4$ from the point $(-1, 1)$ to the point $(1, 1)$.

Solution

$y' = 4x^3$, and the two points correspond to $x = -1$ and $x = 1$, so

$$L = \int_{-1}^1 \sqrt{1 + (4x^3)^2} dx.$$

$$L = \int_{-1}^1 \sqrt{1 + 16x^6} dx$$

3. (6 points) Set up, but **do not evaluate** an integral that computes the surface area of the solid obtained by rotating the curve $y = \cos(x)$ over the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ about the x -axis.

Solution

$y' = -\sin(x)$, and the radius of revolution is $y = \cos(x)$, which is nonnegative on $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

$$S = \int_{-\pi/2}^{\pi/2} 2\pi \cos(x) \sqrt{1 + (-\sin(x))^2} dx.$$

$$S = \int_{-\pi/2}^{\pi/2} 2\pi \cos(x) \sqrt{1 + \sin^2(x)} dx$$

BONUS (2 points) Find the arc length of the curve $y = \sqrt{9 - x^2}$ from the point $(-3, 0)$ to $(0, 3)$. Justify your answer.

Solution

Squaring gives $x^2 + y^2 = 9$ with $y \geq 0$, so the curve is the upper half of the circle of radius 3 centered at the origin. The points $(-3, 0)$ and $(0, 3)$ are a quarter turn apart on that circle, so the arc is one quarter of the full circumference $2\pi(3) = 6\pi$.

$$L = \frac{3\pi}{2}$$