

Name: _____

/ 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. (5 points per part) Evaluate the following integrals and antiderivatives.

(a) $\int_1^e x \ln(x) dx$

Solution

Integrate by parts with $u = \ln(x)$ and $dv = x dx$, so $du = \frac{1}{x} dx$ and $v = \frac{x^2}{2}$:

$$\begin{aligned} \int_1^e x \ln(x) dx &= \left[\frac{x^2}{2} \ln(x) \right]_1^e - \int_1^e \frac{x^2}{2} \cdot \frac{1}{x} dx \\ &= \frac{e^2}{2} - \frac{1}{2} \int_1^e x dx = \frac{e^2}{2} - \frac{1}{2} \left[\frac{x^2}{2} \right]_1^e \\ &= \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4}. \end{aligned}$$

$$\boxed{\int_1^e x \ln(x) dx = \frac{e^2 + 1}{4}}$$

(b) $\int x^2 e^{3x} dx$

Solution

Integrate by parts with $u = x^2$ and $dv = e^{3x} dx$, so $du = 2x dx$ and $v = \frac{e^{3x}}{3}$:

$$\int x^2 e^{3x} dx = \frac{x^2 e^{3x}}{3} - \frac{2}{3} \int x e^{3x} dx.$$

Again with $u = x$ and $dv = e^{3x} dx$, so $du = dx$ and $v = \frac{e^{3x}}{3}$,

$$\int x e^{3x} dx = \frac{x e^{3x}}{3} - \frac{1}{3} \int e^{3x} dx = \frac{x e^{3x}}{3} - \frac{e^{3x}}{9}.$$

Therefore

$$\int x^2 e^{3x} dx = \frac{x^2 e^{3x}}{3} - \frac{2x e^{3x}}{9} + \frac{2e^{3x}}{27} + C.$$

$$\boxed{\int x^2 e^{3x} dx = \frac{e^{3x}}{27} (9x^2 - 6x + 2) + C}$$

2. (5 points per part) Evaluate the following integrals and antiderivatives.

(a) $\int \sin^3(x) \cos^4(x) dx$

Solution

Split off one factor of $\sin(x)$, write $\sin^2(x) = 1 - \cos^2(x)$, and substitute $u = \cos(x)$, $du = -\sin(x) dx$:

$$\begin{aligned} \int \sin^3(x) \cos^4(x) dx &= \int (1 - \cos^2(x)) \cos^4(x) \sin(x) dx \\ &= - \int (1 - u^2) u^4 du = \int (u^6 - u^4) du = \frac{u^7}{7} - \frac{u^5}{5} + C. \end{aligned}$$

$$\boxed{\int \sin^3(x) \cos^4(x) dx = \frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5} + C}$$

(b) $\int \tan^6(x) \sec^4(x) dx$

Solution

Split off one factor of $\sec^2(x)$, write $\sec^2(x) = 1 + \tan^2(x)$, and substitute $u = \tan(x)$, $du = \sec^2(x) dx$:

$$\begin{aligned} \int \tan^6(x) \sec^4(x) dx &= \int \tan^6(x) (1 + \tan^2(x)) \sec^2(x) dx \\ &= \int (u^6 + u^8) du = \frac{u^7}{7} + \frac{u^9}{9} + C. \end{aligned}$$

$$\boxed{\int \tan^6(x) \sec^4(x) dx = \frac{\tan^7(x)}{7} + \frac{\tan^9(x)}{9} + C}$$

(c) $\int_{-\pi/2}^{\pi/2} \cos^3(x) dx$

Solution

Split off one factor of $\cos(x)$ and write $\cos^2(x) = 1 - \sin^2(x)$. With $u = \sin(x)$, $du = \cos(x) dx$, the limits become $u = -1$ and $u = 1$:

$$\int_{-\pi/2}^{\pi/2} \cos^3(x) dx = \int_{-1}^1 (1 - u^2) du = \left[u - \frac{u^3}{3} \right]_{-1}^1 = \frac{2}{3} - \left(-\frac{2}{3} \right).$$

$$\boxed{\int_{-\pi/2}^{\pi/2} \cos^3(x) dx = \frac{4}{3}}$$