Math 252: Quiz 1 **Spring 2024**

Name: Solutions ______ / 24

24 points possible; each part is worth 2 points. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

1. [12 points] Compute the derivatives of the following functions.

a. $f(\theta) = \theta\cos(\theta) + \dfrac{\pi}{2}$

$$f'(\theta) = 1 \cdot \cos(\theta) + \theta(-\sin(\theta)) + 0$$

$$\underline{f'(\theta) = \cos(\theta) - \theta\sin(\theta)}$$

b. $f(x) = 5e^{x/2} + \sin^2(x)$ $= 5e^{\left(\frac{1}{2}x\right)} + (\sin(x))^2$

$$f'(x) = 5 \cdot \frac{1}{2} \cdot e^{\frac{1}{2}x} + 2\sin(x)(\cos(x))$$

$$= \underline{\frac{5}{2}e^{x/2} + 2\sin(x)\cos(x)}$$

c. $h(x) = \sqrt{ax^2 + b^2}$ where a and b are constants

$$h(x) = (ax^2 + b^2)^{1/2}$$

$$h'(x) = \frac{1}{2}(ax^2 + b^2)^{-1/2}(2ax + 0)$$

$$= \boxed{\frac{ax}{\sqrt{ax^2 + b^2}}}$$

d. $f(x) = \ln(\tan(2x) + \sec(2x))$

$$f'(x) = \frac{2\sec^2(2x) + 2\sec(2x)\tan(2x)}{\tan(2x) + \sec(2x)}$$

$$= 2\sec(2x)\left(\frac{\sec(2x) + \tan(2x)}{\tan(2x) + \sec(2x)}\right) = 2\sec(2x)$$

e. $h(x) = \left(x + \sin(x^2+1)\right)^{-2}$

$$h'(x) = -2\left(x + \sin(x^2+1)\right)^{-3}\left(1 + 2x\cos(x^2+1)\right)$$

f. $h(x) = \arctan(x^3)$ $+ \frac{1}{5x} = \arctan(x^3) + \frac{1}{5}x^{-1}$

$$h'(x) = \frac{3x^2}{1+(x^3)^2} - \frac{1}{5}x^{-2} = \frac{3x^2}{1+x^6} - \frac{1}{5}x^{-2}$$

2. **[12 points]** Compute the following antiderivatives (indefinite integrals) and definite integrals. Remember that antiderivatives need a "$+C$".

a. $\int_{-1}^{1} x(2-x)\,dx = \int_{-1}^{1} (2x - x^2)\,dx = x^2 - \frac{1}{3}x^3\Big]_{-1}^{1}$

$= \left(1^2 - \frac{1}{3}(1)^3\right) - \left((-1)^2 - \frac{1}{3}(-1)^3\right)$

$= \left(1 - \frac{1}{3}\right) - \left(1 + \frac{1}{3}\right) = \boxed{\frac{-2}{3}}$

b. $\int \sin(\pi x) + \frac{2}{3x}\,dx = \int \left(\sin(\pi x) + \frac{2}{3}x^{-1}\right) dx$

$= -\frac{1}{\pi}\cos(\pi x) + \frac{2}{3}\ln|x| + C$

c. $\int \frac{x}{\sqrt{2+x^2}}\,dx = \int x(2+x^2)^{-1/2}\,dx$

$u = 2 + x^2$

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

$= \frac{1}{2}\int u^{-1/2}\,du$

$= -u^{\frac{1}{2}} + C$

$= -\sqrt{2+x^2} + C$

d. $\int_0^{\pi/2} cos(x)(\sin(x)+1)^3\, dx$

$$\begin{cases} u = \sin(x)+1 \\ du = \cos(x)\,dx \\ \text{If } x=0,\ u=1 \\ x=\pi/2,\ u=2 \end{cases}$$

$$= \int_1^2 u^3\,du = \frac{1}{4}u^4\Big]_1^2$$

$$= \frac{1}{4}(2^4 - 1^4) = \boxed{\frac{15}{4}}$$

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e. $\int \frac{e^x}{1+e^{2x}}\, dx = \int \frac{e^x}{1+(e^x)^2}\,dx$

$$\begin{cases} \text{let } u = e^x \\ du = e^x\,dx \end{cases}$$

$$= \int \frac{du}{1+u^2}$$

$$= \arctan(u) + C = \underline{\arctan(e^x) + C}$$

f. $\int \frac{x}{(x+1)^2}\, dx = \int x(x+1)^{-2}\,dx$

$$\begin{cases} u = x+1 \\ du = dx \\ x = u-1 \end{cases}$$

$$= \int (u-1)\cdot u^{-2}\,du$$

$$= \int (u^{-1} - u^{-2})\,du = \ln|u| + u^{-1} + C$$

$$= \underline{\ln|x+1| + (x+1)^{-1} + C}$$