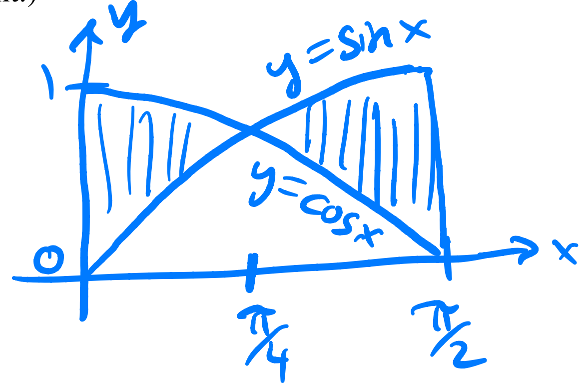
Math 252 (Bueler): Quiz 2 **25 January 2024**

Name: SOLUTIONS

________ / 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. **[5 points]** Find the area of the region between $y = \sin x$ and $y = \cos x$ on the interval $[0, \pi/2]$. (*Hint: Draw a careful sketch first! You may use symmetry if you want.*)



$$A = \int_0^{\pi/2} |\sin x - \cos x| \, dx$$

$$= 2 \int_0^{\pi/4} \cos x - \sin x \, dx$$

uses symmetry

$$= 2 \left[\sin x + \cos x \right]_0^{\pi/4}$$

$$= 2 \left[\left(\tfrac{1}{\sqrt{2}} + \tfrac{1}{\sqrt{2}} \right) - (0 + 1) \right]$$

$$= \frac{4}{\sqrt{2}} - 2 = 2\sqrt{2} - 2 = 2(\sqrt{2} - 1)$$

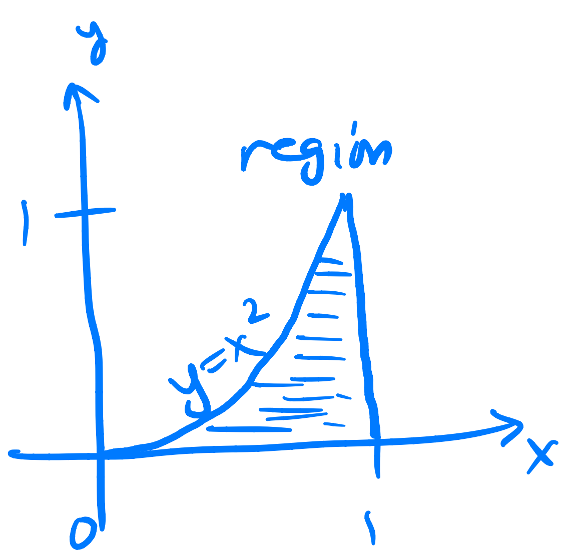
← any of these are simplified enough

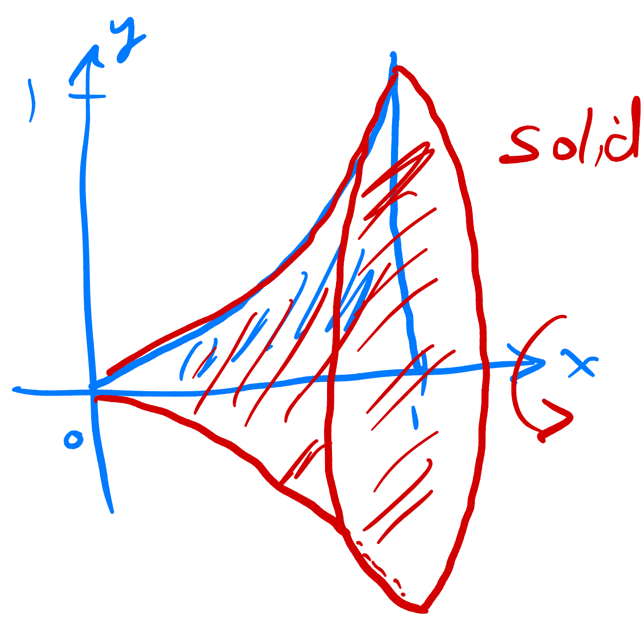
$\sin x = \cos x$

$x = \pi/4$

2. **[15 points]**

   a. Sketch the region bounded by $y = x^2$, $y = 0$, and $x = 1$. Then sketch the solid of revolution formed by rotating the region around the x-axis. **Please make your sketches adequately large and clear!**



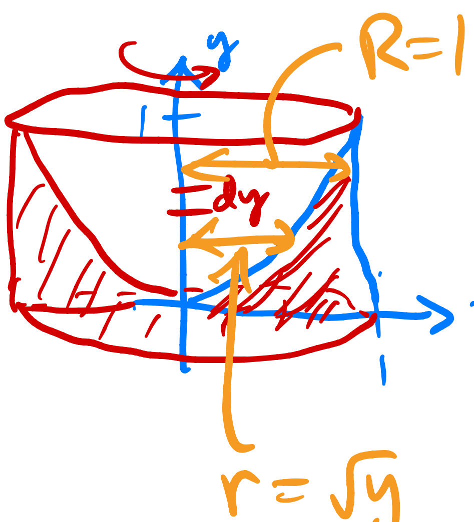


b. Find the volume of the solid which you sketched in part **a**. (*Hint: Use discs or washers.*)

$$V = \int_0^1 \pi \underbrace{r}_{\text{at position } x\text{, radius is } y\text{-value}}{}^2 \, dx = \int_0^1 \pi (x^2)^2 \, dx$$

$$= \pi \int_0^1 x^4 \, dx = \pi \left[\frac{x^5}{5} \right]_0^1 = \frac{\pi}{5}$$

c. Find the volume of the solid formed by revolving the region in part **a** around the y-axis. (*Hint: Sketch the solid. Use discs or washers.*)



$[y = x^2 \Leftrightarrow x = \sqrt{y}]$

$$V = \int_0^1 \pi (\underbrace{R}^2 - \underbrace{r}^2) \, dy$$

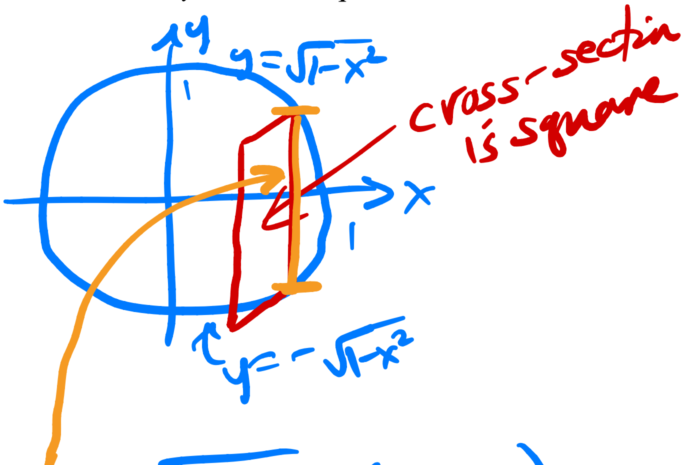
these are x-values and functions of y

$$= \int_0^1 \pi \left(1 - (\sqrt{y})^2\right) dy$$

$$= \pi \int_0^1 1 - y \, dy = \pi \left[y - \frac{y^2}{2} \right]_0^1$$

$$= \pi \left(1 - \frac{1}{2}\right) = \frac{\pi}{2}$$

3. [5 points] A solid has a base which is the unit circle in the x, y plane, and each cross-section parallel to the y-axis is a square. Find the volume.



$$V = \int_{-1}^{1} A(x)\,dx$$

area of red square

$$s = \sqrt{1-x^2} - \left(-\sqrt{1-x^2}\right)$$

$$= 2\sqrt{1-x^2}$$

$$= \int_{-1}^{1} \left(2\sqrt{1-x^2}\right)^2 dx$$

$$= 4\int_{-1}^{1} 1-x^2\,dx$$

optional: use symmetry

$$= 8\int_{0}^{1} 1-x^2\,dx$$

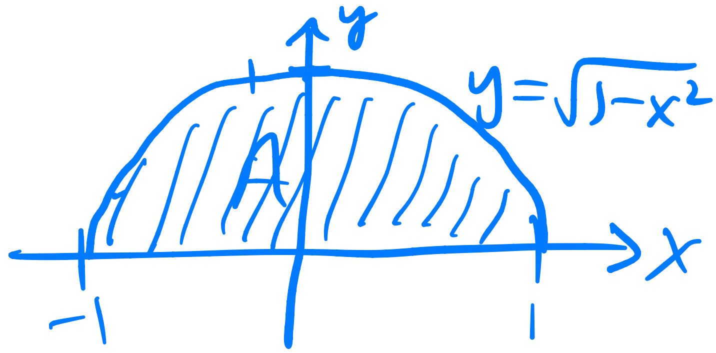
$$= 8\left[x - \frac{x^3}{3}\right]_0^1 = 8\left(1-\frac{1}{3}\right) = \frac{16}{3}$$

EC. [1 points] (**Extra Credit**) Give the correct value of the definite integral:

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx.$$

(**Hint.** There is no requirement to use the fundamental theorem of calculus. What is sought is the correct answer, **with some justification**, which might be a sketch.)

Sketch:



$$\int_{-1}^{1} \sqrt{1-x^2}\,dx = A = (\text{area of semicircle})$$

$$= \frac{1}{2}\pi r^2 = \boxed{\frac{\pi}{2}}$$

$r=1$

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