Math 252 (Bueler): Quiz 5 **15 February 2024**

Name: SOLUTIONS

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30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. [18 points] Compute the following integrals.

a. $\int x e^{-x}\,dx = x(-e^{-x}) - \int (-e^{-x})\,dx = -xe^{-x} + \int e^{-x}\,dx$

$\left[\begin{array}{ll} u = x & v = -e^{-x} \\ du = dx & dv = e^{-x}\,dx \end{array}\right]$

$= -xe^{-x} - e^{-x} + C$

b. $\int_1^3 x \ln x\,dx = \left[(\ln x)\frac{x^2}{2}\right]_1^3 - \int_1^3 \frac{x^2}{2}\frac{dx}{x}$

$\left[\begin{array}{ll} u = \ln x & v = x^2/2 \\ du = \frac{dx}{x} & dv = x\,dx \end{array}\right]$

$= \frac{9}{2}\ln 3 - 0 - \frac{1}{2}\int_1^3 x\,dx = \frac{9}{2}\ln 3 - \frac{1}{2}\left[\frac{x^2}{2}\right]_1^3$

$= \frac{9}{2}\ln 3 - \frac{1}{4}(9-1) = \frac{9}{2}\ln 3 - 2$

c. $\int \cos x\, e^{-\sin x}\,dx = \int e^{-u}\,du = -e^{-u} + C$

$\left[\begin{array}{l} u = \sin x \\ du = \cos x\,dx \end{array}\right]$

$= -e^{-\sin x} + C$

d. $\int \cos^4 w \sin^3 w \, dw = \int \cos^4 w \, (1-\cos^2 w) \sin w \, dw$

$$= -\int u^4 (1-u^2)\, du = \int u^6 - u^4 \, du$$

$\left[\begin{array}{l} u = \cos w \\ -du = \sin w \, dw \end{array}\right]$

$$= \frac{1}{7} u^7 - \frac{1}{5} u^5 + C$$

$$= \frac{1}{7}\cos^7 w - \frac{1}{5}\cos^5 w + C$$

e. $\int \tan^2 x \sec^2 x \, dx = \int u^2 \, du = \frac{1}{3} u^3 + C$

$\left[\begin{array}{l} u = \tan x \\ du = \sec^2 x \, dx \end{array}\right]$

$$= \frac{1}{3}\tan^3 x + C$$

f. $\underbrace{\int e^x \sin x \, dx}_{= I} = e^x(-\cos x) - \int (-\cos x)\, e^x \, dx$

$\left[\begin{array}{ll} u = e^x & v = -\cos x \\ du = e^x dx & dv = \sin x \, dx \end{array}\right]$

$$= -e^x \cos x + \int e^x \cos x \, dx$$

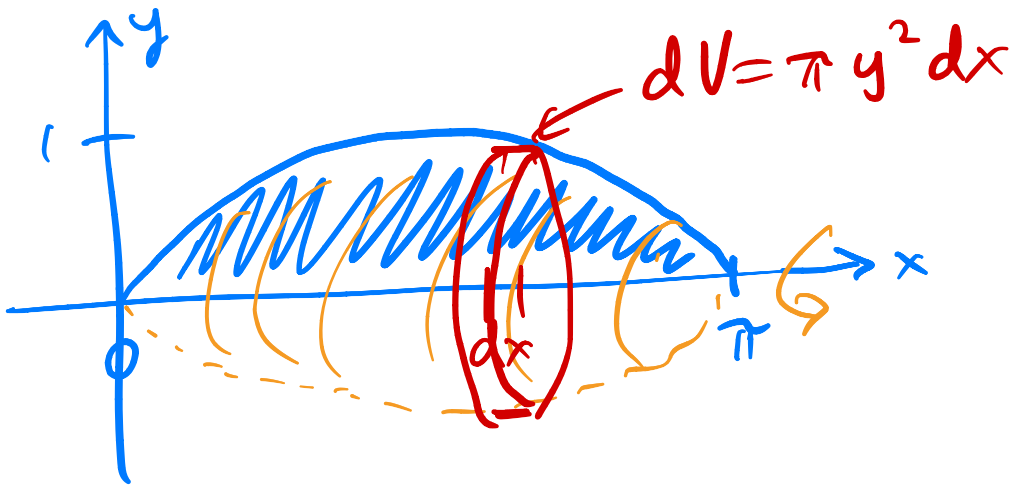
$\left[\begin{array}{ll} u = e^x & v = \sin x \\ du = e^x dx & dv = \cos x \, dx \end{array}\right]$

$$= -e^x \cos x + \left[e^x \sin x - \int \sin x \, e^x \, dx \right]$$

$$= -e^x \cos x + e^x \sin x - I$$ so $2I = e^x(-\cos x + \sin x)$

$$I = \frac{1}{2} e^x(-\cos x + \sin x) + C$$

2. **[7 points]** Sketch the region between $y = \sin x$ and the x-axis on the interval $0 \le x \le \pi$. Find the volume of the solid which results by rotating the region around the x-axis. (*Hint. Use disks.*)



$$V = \int_0^{\pi} \pi \sin^2 x \, dx$$

$$= \pi \int_0^{\pi} \frac{1 - \cos(2x)}{2} \, dx$$

$$= \frac{\pi}{2} \left[x - \frac{1}{2}\sin(2x) \right]_0^{\pi}$$

$$= \frac{\pi^2}{2}$$

Extra Credit. [1 point] Assume n is a large integer. One of these indefinite integrals is much easier than the other. Circle the **easier** one, and do it.

$$\int \sec^n x \tan x \, dx \qquad\qquad \int \tan^n x \sec x \, dx$$

$$= \int \sec^{n-1} x \sec x \tan x \, dx = \int u^{n-1} du$$

$$\left[\begin{array}{l} u = \sec x \\ du = \sec x \tan x \, dx \end{array}\right]$$

$$= \frac{u^n}{n} + C$$

$$= \frac{1}{n} \sec^n x + C$$

You may find the following **trigonometric formulas** useful. Other formulas, not listed here, should be in your memory, or you can derive them from the ones here.

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$$
$$\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$$
$$\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$$

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