Math 252 (Bueler): Quiz 7 **21 March 2024**

Name: ______________________________ | ________ / 25

30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. [3 points] Find a formula for the nth term a_n of the sequence whose first several terms are

$$0, 3, 8, 15, 24, 35, 48, 63, 80, 99, \dots$$

2. [5 points] Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

a. Use partial fractions and “telescoping” to write a simplified formula for the partial sum S_k.

b. Compute the sum of the infinite series.

3. [6 points] For each sequence, find the limit if it converges, or show that the sequence diverges. Indicate any places where you use L'Hôpital's rule.

a. $\dfrac{n^2}{2^n}$

b. $a_n = \left(1 - \dfrac{2}{n}\right)^n$

4. [3 points] Find a formula for the nth term a_n of this recursively-defined sequence, for $n \geq 1$:

$$a_1 = 3 \quad \text{and} \quad a_{n+1} = \frac{a_n}{n}.$$

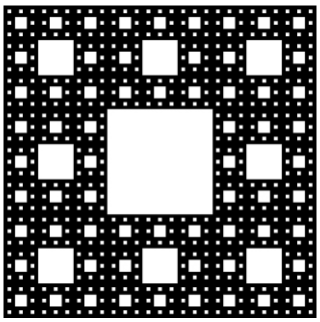
5. [6 points] Does the series converge or diverge? If it converges find its sum; if it diverges explain why.

a. $1+\frac{e}{\pi}+\frac{e^2}{\pi^2}+\frac{e^3}{\pi^3}+\dots$

b. $\sum_{n=1}^{\infty}\frac{n+1}{n}$

6. [2 points] Compute and simplify the partial sum S_4 for the series in **5 b.**

Extra Credit. [1 point] The thing below is called the **Sierpinksi gasket**. It is built by removing the white parts from a black square. Assume the original square has side-length one and thus area one. Remove the middle 1/9th of the area. The remainder is 8 smaller black squares. For each of these, remove the middle 1/9th. Continuing in this way forever, you remove all the white area. Using geometric series, compute the white area you removed. What area is left, the black area?



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