Math 252: Quiz 10 **23 Apr 2026**

Name: SOLUTIONS

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30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. [8 points] Write the MacLaurin Series of each function.

a. $f(x) = (1+x^2)^{1/3}$ ← binomial series

$$f(x) = \sum_{n=0}^{\infty} \binom{1/3}{n} (x^2)^n$$

$$= \sum_{n=0}^{\infty} \binom{1/3}{n} x^{2n}$$

b. $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$. Express your answer as **one series**.

$$\cosh(x) = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

write series for each

$$= \frac{1}{2}\left(\sum_{n=0}^{\infty} \frac{x^n}{n!} + \sum_{n=0}^{\infty} \frac{(-x)^n}{n!}\right)$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2} + \frac{(-1)^n}{2}\right) \frac{x^n}{n!} = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$$

$= 1$ if n even
$= 0$ if n odd

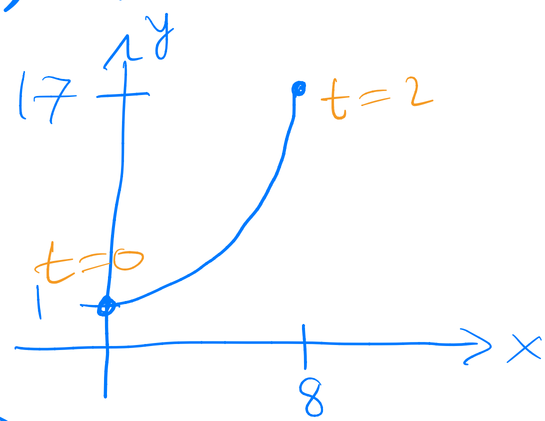
2. [8 points] Consider the parametric equations $x = 2t^2, \quad y = t^4 + 1, \quad t \in [0, 2]$.

a. Eliminate the parameter and sketch the graph.

$t^2 = \frac{x}{2}$ $\therefore$ $t = \left(\frac{x}{2}\right)^{1/2}$

$\therefore$ $y = \left(\left(\frac{x}{2}\right)^{1/2}\right)^4 + 1 = \left(\frac{x}{2}\right)^2 + 1$

$y = \frac{1}{4}x^2 + 1$



b. Find $\frac{d^2y}{dx^2}$ using the result of part **a**.

$\frac{dy}{dx} = \frac{1}{2}x$ $\therefore$ $\frac{d^2y}{dx^2} = \frac{1}{2}$

c. Find $\frac{d^2y}{dx^2}$ using the parametric formula, and check that it gives the same result as in part **b**.

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$= \frac{\frac{d}{dt}(t^2)}{4t} = \frac{2t}{4t} = \frac{1}{2}$$

✓

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t^3 + 0}{4t} = t^2$$

3. [9 points] Consider the parametric equations $x = t - \sin t, \quad y = 1 - \cos t, \quad t \in [0, 2\pi]$.

a. Find the equation of the tangent line at $t = \frac{\pi}{2}$. ← will need point-slope: $y - y_0 = m(x - x_0)$

$$x_0 = x(\pi/2) = \frac{\pi}{2} - \cancel{\sin(\pi/2)}^{1} = \pi/2 - 1$$

$$y_0 = y(\pi/2) = 1 - \cancel{\cos(\pi/2)}^{0} = 1$$

$$m = \frac{dy}{dx}\Big|_{t=\pi/2} = \frac{0 + \sin t}{1 - \cos t}\Big|_{t=\pi/2} = \frac{\sin(\pi/2)}{1 - \cos(\pi/2)} = \frac{1}{1-0} = 1$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\therefore \quad y - 1 = 1 \cdot \left(x - \left(\frac{\pi}{2} - 1\right)\right)$$

or: $y = x - \frac{\pi}{2} + 2$

b. Set up (but do not evaluate) an integral for the area under the parametric curve.

$$A = \int_0^{2\pi} y(t)\, x'(t)\, dt = \int_0^{2\pi} (1 - \cos t)(1 - \cos t)\, dt$$

$$= \int_0^{2\pi} (1 - \cos t)^2\, dt$$

c. Set up (but do not evaluate) an integral for the arc length of the parametric curve.

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(1 - \cos t)^2 + (\sin t)^2}\, dt$$

Extra Credit. [1 point] Evaluate the integral in **Problem 3 c.**

$$L = \int_0^{2\pi} \sqrt{(1-\cos t)^2 + \sin^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt = \sqrt{2} \int_0^{2\pi} \sqrt{1-\cos t}\, dt$$

$$\left[\sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta) \quad \therefore \quad 1-\cos t = 2\sin^2(t/2)\right]$$

$$= \sqrt{2} \int_0^{2\pi} \sqrt{2\sin^2(t/2)}\, dt = 2\int_0^{2\pi} \sin(t/2)\, dt$$

$$= 2\left[-2\cos(t/2)\right]_0^{2\pi} = -4(\cos(\pi) - \cos(0))$$

$$= 8$$

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