Math 252: Quiz 2 **22 January 2026**

Name: SOLUTIONS

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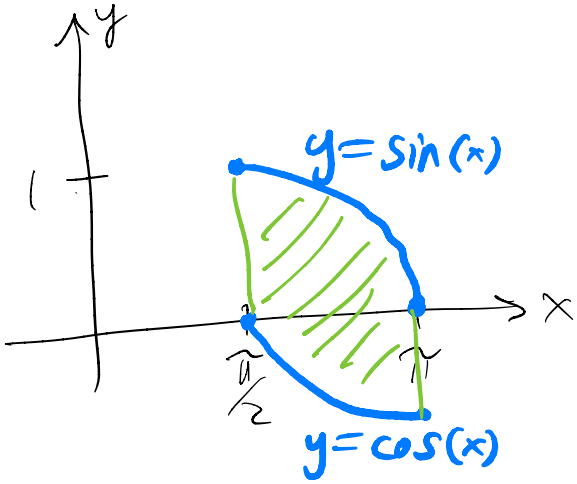
30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. **Please make your sketches large and clear!** 25 points possible.

1. [5 points] Find the area of the region between $y = \cos x$ and $y = \sin x$ on the interval $[\pi/2, \pi]$. (*Hint: Draw a careful sketch first!*)



$$A = \int_{\pi/2}^{\pi} \sin(x) - \cos(x)\, dx$$

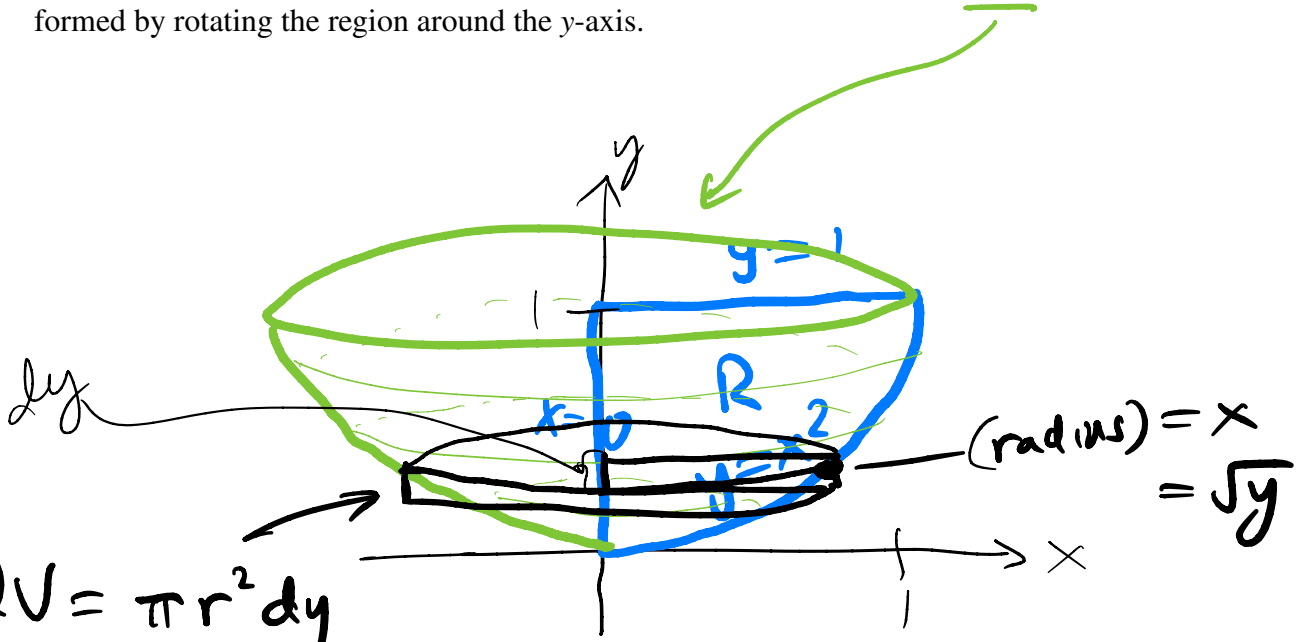
$$= -\cos(x) - \sin(x) \Big]_{\pi/2}^{\pi}$$

$$= (-(-1) - 0) - (-0 - 1)$$

$$= 1 + 1 = 2$$

2. [15 points]

a. Sketch the region bounded by $y = x^2$, $y = 1$, and $x = 0$. Then sketch the solid of revolution formed by rotating the region around the y-axis.



$$dV = \pi r^2 dy$$

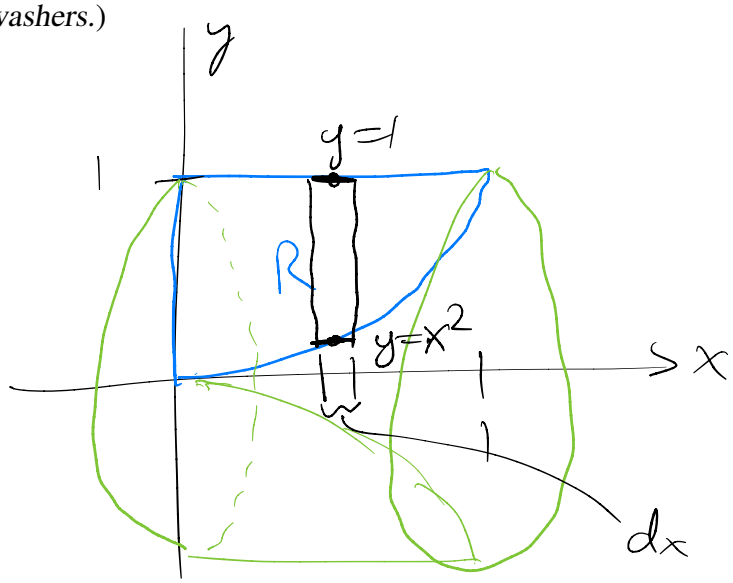
$$= \pi (\sqrt{y})^2 dy$$

b. Find the volume of the solid which you sketched in part **a**, from the previous page. (*Hint: Use discs or washers.*)

$$V = \int_0^1 \pi \left(\underbrace{\sqrt{y}}_{\text{radius of disc}}\right)^2 dy$$

$$= \int_0^1 \pi y \, dy = \pi \frac{y^2}{2} \Big]_0^1 = \boxed{\frac{\pi}{2}}$$

c. Find the volume of the solid formed by revolving the region in part **a** around the x-axis. (*Hints: Sketch the solid. Then use discs or washers.*)



$$V = \int_0^1 \pi (1^2 - (x^2)^2) \, dx$$

$$= \pi \int_0^1 1 - x^4 \, dx$$

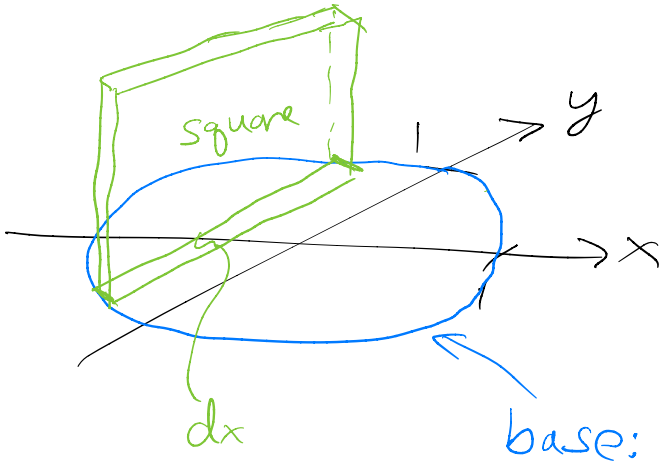
$$= \pi \left[x - \frac{x^5}{5}\right]_0^1$$

$$= \pi \left[1 - \frac{1}{5}\right] = \boxed{\frac{4\pi}{5}}$$

3. [5 points] A solid has a base which is the unit circle in the x, y plane, and each cross-section which is perpendicular to the base and parallel to the y-axis is a square. Find the volume. (*Hint: A sketch is a good idea, of course, but what you really want is to understand the side length of the square!*)

$$V = \int_{-1}^{1} \left(\sqrt{1-x^2} - \left(-\sqrt{1-x^2}\right)\right)^2 dx$$

area of square



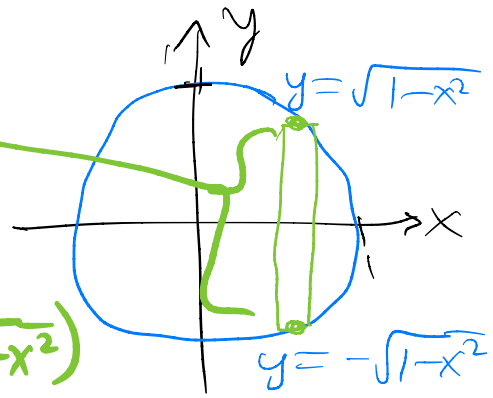
$$= \int_{-1}^{1} \left(2\sqrt{1-x^2}\right)^2 dx$$

$$= 4\int_{-1}^{1} 1-x^2\,dx$$

even function

$$= 8\int_{0}^{1} 1-x^2\,dx$$

length $= \sqrt{1-x^2} - \left(-\sqrt{1-x^2}\right)$



$$= 8\left[x - \frac{x^3}{3}\right]_0^1 = 8\left[1 - \frac{1}{3}\right]$$

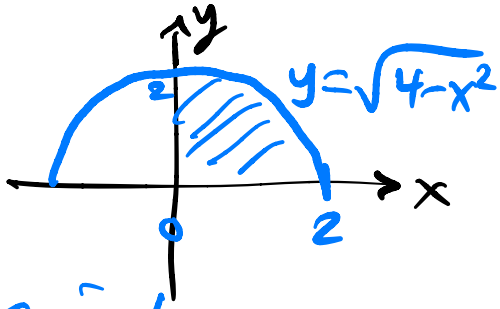
$$= \frac{16}{3}$$

EC. [1 points] (**Extra Credit**) Give the correct final value of this definite integral:

$$\int_0^2 \sqrt{4-x^2}\,dx.$$

(**Hint.** There is no requirement to use the fundamental theorem of calculus! You want to provide the correct answer, **with a justification**. Your justification might be a sketch.)

the integrand is the upper semi-circle of radius 2, so the integral is $\frac{1}{4}$ of the area of a circle:



$$\int_0^2 \sqrt{4-x^2}\,dx = \frac{1}{4}(\pi \cdot 2^2) = \pi$$

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