Math 252: Quiz 3 **29 January 2026**

Name: SOLUTIONS

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30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. **[8 points]** For each part below, completely set up, **but do not evaluate**, the integral.

a. The length of the curve $y = \sin(x^2) + 2$ on the interval $2 \le x \le 4$.

$$f'(x) = \cos(x^2) \cdot 2x$$

$$L = \int_2^4 \sqrt{1 + (\cos(x^2)\, 2x)^2}\, dx$$

$$= \int_2^4 \sqrt{1 + 4x^2 \cos^2(x^2)}\, dx$$

b. The area of the surface formed by revolving the graph of $y = x^3 + 2$, on the interval $[0,2]$, around the x-axis.

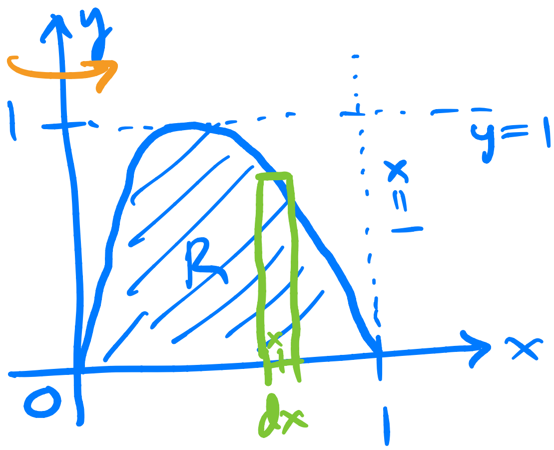
$$f'(x) = 3x^2$$

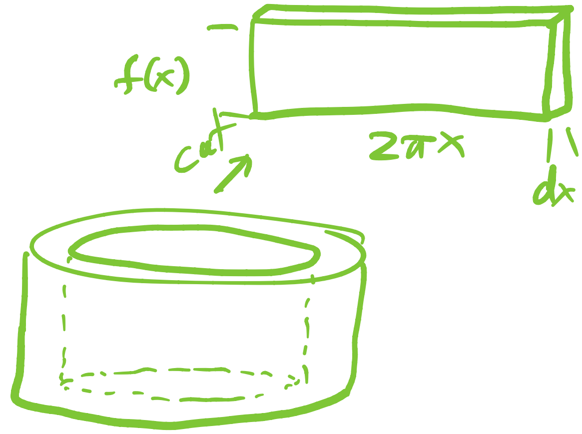
$$A = \int_0^2 2\pi (x^3+2) \sqrt{1 + (3x^2)^2}\, dx$$

$$= 2\pi \int_0^2 (x^3+2) \sqrt{1 + 9x^4}\, dx$$

2. [8 points]

a. Sketch the region bounded by the curves $y = \sin(\pi x^2)$, $y = 0$, $x = 0$, and $x = 1$.





b. Set up **and evaluate** an integral to compute the volume of the solid found by rotating the region in **a.** around the y-axis. (*Hint. The integral from using washers won't work. Using shells you can do the integral.*)

$$V = \int_0^1 2\pi x \cdot \sin(\pi x^2) \cdot dx$$

$$u = \pi x^2$$
$$du = 2\pi x\, dx$$

$$= \int_0^{\pi} \sin(u)\, du$$

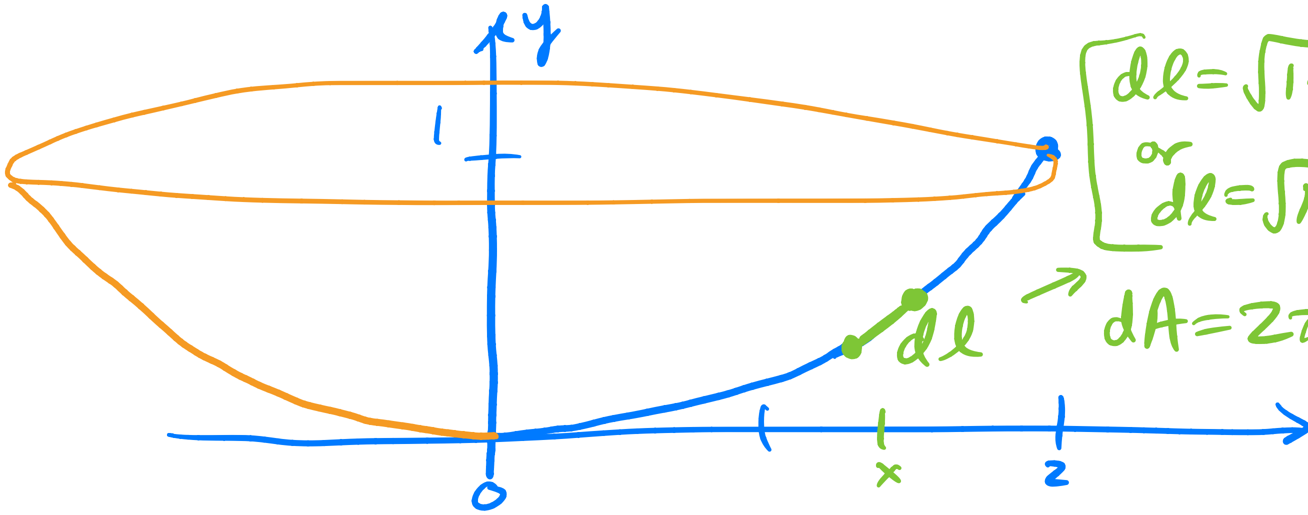
$$= -\cos(u)\Big]_0^{\pi} = -\cos(\pi) + \cos(0)$$

$$= -(-1) + 1 = 2$$

3. [9 points] A team of engineers needs to build a large parabolic radio antenna, a satellite dish like those on West Campus. Their design has a radius of 2 m and a depth of 1 m. They need to know the surface area to determine how much material is needed to build one.

same as: $x = 2\sqrt{y}$

a. Rotate the curve $y = \dfrac{x^2}{4}$, $0 \le x \le 2$, around the y-axis to create the surface. Sketch the curve and the surface.



$$\boxed{\begin{aligned} d\ell &= \sqrt{1+\left(\tfrac{dy}{dx}\right)^2}\,dx \\ &\text{or} \\ d\ell &= \sqrt{1+\left(\tfrac{dx}{dy}\right)^2}\,dy \end{aligned}}$$

$$dA = 2\pi x \cdot d\ell$$

b. Set up **and evaluate** an integral to compute the surface area. Simplify your answer. (*Hint. Yes, you can do the integral!*)

$$A = \int_0^2 2\pi x \sqrt{1+\left(\tfrac{x}{2}\right)^2}\,dx$$

$[u = 1+\tfrac{x^2}{4},\ du = \tfrac{1}{2}x\,dx]$

$$= 2\pi \int_0^2 x\left(1+\tfrac{1}{4}x^2\right)^{1/2} dx = 2\pi \int_1^2 u^{1/2} \cdot 2\,du$$

$$= 4\pi \left[\tfrac{2}{3}u^{3/2}\right]_1^2 = \frac{8\pi}{3}\left(2^{3/2}-1\right)$$

BOTH are correct!

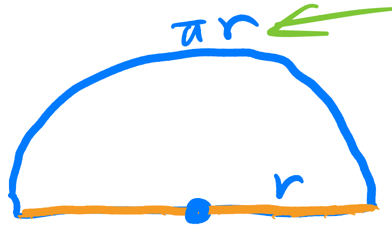
$$A = \int_0^1 2\pi(2\sqrt{y})\sqrt{1+\left(y^{-1/2}\right)^2}\,dy$$

$$= 4\pi \int_0^1 \sqrt{y}\sqrt{1+\tfrac{1}{y}}\,dy = 4\pi\int_0^1 \sqrt{y+1}\,dy$$

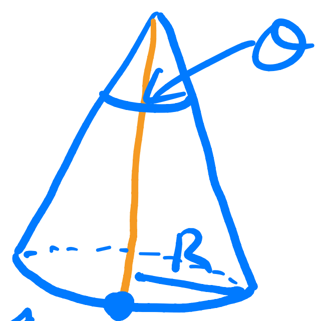
$$= 4\pi\, \tfrac{2}{3}(y+1)^{3/2}\Big]_0^1 = \frac{8\pi}{3}\left(2^{3/2}-1\right)$$

EC. [1 points] (**Extra Credit**) If you draw half of a disk on a piece of paper, and cut that out, and roll it into a cone, what's the angle at the point of the cone? (*Hint. No integration needed! Use your geometry and triangle knowledge!*)

half of a circle

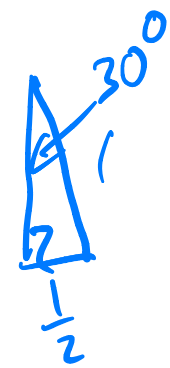


roll up



$\frac{r}{2} = R$

base circle: $2\pi R$



$2\pi R = \pi r$

$\therefore\ R = \frac{r}{2}$

$\frac{\theta}{2} = 30^\circ$

$\theta = 60^\circ$

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