Math 252: Quiz 4 **5 February 2026**

Name: _____ _____ / 25

30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. **[3 points]** Compute the total work done by a force $F(x) = cos(\pi x)$ on an object which moves from $x = 0$ to $x = \frac{1}{2}$.

$$\int_0^{1/2} \cos(\pi x)\, dx = \left[\frac{1}{\pi}\sin(\pi x)\right]_0^{1/2} = \frac{1}{\pi}\sin(\pi/2) - \frac{1}{\pi}\sin(0)$$

$$= \frac{1}{\pi} - 0$$

$$= \frac{1}{\pi} \text{ J}$$

2. **[6 points]** A 1-dimensional rod is 3 meters long and has linear mass density $\rho(x) = 1 + \frac{2}{(1+x)^2}$ kilograms per meter (starting at $x = 0$). Find its mass. Simplify your answer and include units.

$$\int_0^3 1 + \frac{2}{(1+x)^2}\, dx$$

$u = 1 + x$

$du = dx$

$$= \int_1^4 \left(1 + 2u^{-2}\right) du$$

$$= \left[u - 2u^{-1}\right]_1^4$$

$$= \left(4 - \frac{2}{4}\right) - \left(1 - \frac{2}{1}\right)$$

$$= \frac{7}{2} - (-1)$$

$$= \frac{9}{2} \text{ kg}$$

3. [8 points] A spring has natural length of 3 meters. It requires 6 J of work to stretch the spring from its natural length to a length of 4 meters. How much work would it take to stretch the spring from 4 meters to 5 meters?

(Hooke's law: $F(x) = kx$)

6 J to stretch from 3m to 4m:

$$6 = \int_0^1 kx\,dx = \left[\tfrac{1}{2}kx^2\right]_0^1 = \tfrac{1}{2}k$$

$$\longrightarrow k = 12 \text{ N/m}$$

To stretch from 4m to 5m:

$$\int_1^2 12x\,dx = \left[6x^2\right]_1^2 = 6\cdot 4 - 6\cdot 1$$

$$= 24 - 6$$

$$\boxed{= 18 \text{ J}}$$

4. [8 points] Find the derivative, indefinite integral, or definite integral. Put your answers in reasonably simplified form and write "$+C$" if appropriate.

a. Find $\dfrac{dy}{dx}$ if $y = \dfrac{\ln(x^2)}{x}$.

$$\frac{dy}{dx} = \frac{\left(\frac{1}{x^2}\cdot 2x\right)\cdot x - \ln(x^2)(1)}{x^2} = \boxed{\frac{2-\ln(x^2)}{x^2}}$$

b. $\displaystyle\int_0^{\pi/4} \tan x\,dx =$ $\displaystyle\int_0^{\pi/4} \frac{\sin x}{\cos x}\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$$= \int_1^{\sqrt{2}/2} -\frac{1}{u}\,du$$

$$= \left[-\ln u\right]_1^{\sqrt{2}/2}$$

$$= \boxed{-\ln\frac{\sqrt{2}}{2} = -\ln\frac{1}{\sqrt{2}} = \ln\sqrt{2} = \frac{1}{2}\ln 2}$$

c. Find $\dfrac{dy}{dx}$ if $y = 2^{5x} + \log_2(5x)$.

$$\frac{dy}{dx} = 2^{5x}\ln(2)\cdot 5 + \frac{1}{5x\cdot\ln(2)}\cdot 5$$

$$= \boxed{2^{5x}(5\ln(2)) + \frac{1}{x\cdot\ln(2)}}$$

d. $\displaystyle\int \frac{dx}{x\ln x} =$ $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \boxed{\ln|\ln x| + C.}$$

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