Math 252: Quiz 5 **12 February 2026**

Name: _____ / **25**

30 minutes maximum. No aids (internet, book, etc.) are permitted, but **please see the trigonometric identities on the final page!** For full credit please show all work, use proper notation, and put answers in reasonably-simplified form. 25 points possible.

1. [15 points] Evaluate the following integrals.

a. $\int x e^{2x}\, dx =$

b. $\int \cos^3 w \sin^4 w\, dw =$

c. $\int_0^1 \arctan x\, dx =$

d. $\int e^x \sin x \, dx =$

e. $\int \tan^2 x \, dx =$

2. [4 points] We say two functions $f(x), g(x)$ are *orthogonal* on the interval $[-\pi, \pi]$ if the integral of their product is zero: $\int_{-\pi}^{\pi} f(x)\, g(x)\, dx = 0$. Show that the functions $\sin(2x)$ and $\cos(3x)$ are orthogonal on the interval $[-\pi, \pi]$.

3. [6 points] Sketch the region between $y = \cos x$ and the x-axis on the interval $0 \leq x \leq \pi/2$. Find the volume of the solid which results by rotating the region around the x-axis. (*Hint. Use disks.*)

EC. [1 points] (**Extra Credit**) Assume n is a large and positive integer. One of these indefinite integrals is much easier than the other. **Circle the <u>easier</u> one, and then evaluate it.**

$$\int \tan^n x \sec x \, dx \qquad\qquad \int \sec^n x \tan x \, dx$$

You may find the following **trigonometric formulas** useful. Other formulas, not listed here, should be in your memory, or you can derive them from the ones here.

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$$

$$\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$$

$$\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$$

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