Math 252: Quiz 5 **12 February 2026**

Name: SOLUTIONS

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30 minutes maximum. No aids (internet, book, etc.) are permitted. Please show all work, use proper notation, and put answers in reasonably-simplified form for full credit. 25 points possible.

1. [15 points] Evaluate the following integrals.

a. $\int xe^{2x}\,dx = x\cdot\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}\,dx$

$\left[\begin{matrix} u=x & v=\frac{1}{2}e^{2x} \\ du=dx & dv=e^{2x}dx \end{matrix}\right]$

$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$

$= \boxed{\frac{1}{2}e^{2x}\left(x-\frac{1}{2}\right)+C}$

b. $\int \cos^3 w\sin^4 w\,dw = \int \sin^4 w\cos^2 w\,\cos w\,dw$

$= \int \sin^4 w\,(1-\sin^2 w)\cos w\,dw$ ← $u=\sin w$

$du = \cos w\,dw$

$= \int u^4(1-u^2)\,du = \int u^4 - u^6\,du$

$= \frac{1}{5}u^5 - \frac{1}{7}u^7 + C = \boxed{\frac{1}{5}\sin^5 w - \frac{1}{7}\sin^7 w + C}$

c. $\int_0^1 \arctan x\,dx = x\arctan x\Big]_0^1 - \int_0^1 x\frac{dx}{1+x^2}$

$\left[\begin{matrix} u=\arctan x & v=x \\ du=\frac{dx}{1+x^2} & dv=dx \end{matrix}\right]$

$w = 1+x^2$

$\frac{dw}{2} = x\,dx$

$= 1\cdot\arctan(1) - 0\cdot\overset{0}{\cancel{\arctan(0)}} - \int_1^2 \frac{dw/2}{w}$

$= \arctan(1) - \frac{1}{2}\ln|w|\Big]_1^2 = \boxed{\frac{\pi}{4} - \frac{\ln 2}{2}}$

d. $\int e^x \sin x\,dx = e^x(-\cos x) - \int(-\cos x)e^x dx$

$\begin{bmatrix} u = e^x & v = -\cos x \\ du = e^x dx & dv = \sin x dx \end{bmatrix}$

$= -e^x \cos x + \int e^x \cos x dx$

$\begin{bmatrix} u = e^x & v = \sin x \\ du = e^x dx & dv = \cos x dx \end{bmatrix}$

$= -e^x \cos x + \left(e^x \sin x - \int \sin x\, e^x dx\right)$

$I = e^x(\sin x - \cos x) - I \iff 2I = e^x(\sin x - \cos x)$

$\therefore \quad \int e^x \sin x dx = \boxed{\frac{1}{2} e^x(\sin x - \cos x) + C}$

e. $\int \tan^2 x\,dx =$

$\int \sec^2 x - 1 dx = \boxed{\tan x - x + C}$

2. **[4 points]** We say two functions $f(x), g(x)$ are *orthogonal* on the interval $[-\pi, \pi]$ if the integral of their product is zero: $\int_{-\pi}^{\pi} f(x)\,g(x)\,dx = 0$. Show that the functions $\sin(2x)$ and $\cos(3x)$ are orthogonal on the interval $[-\pi, \pi]$.

$\int_{-\pi}^{\pi} \sin(2x)\cos(3x)\,dx$ ⟵ from last page: $\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$

$= \frac{1}{2}\int_{-\pi}^{\pi} \sin(-x) + \sin(5x)\,dx$

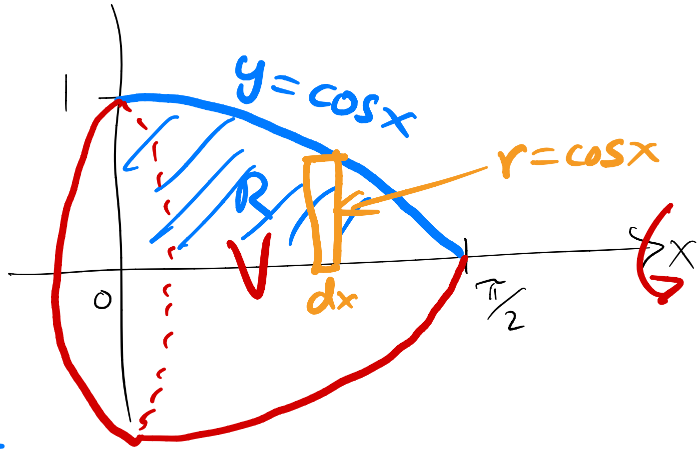
$= \frac{1}{2}\int_{-\pi}^{\pi} -\sin x + \sin(5x)\,dx$

$= \frac{1}{2}\left[\cos x - \frac{1}{5}\cos(5x)\right]_{-\pi}^{\pi} = \frac{1}{2}\left(\cancel{\cos\pi} - \frac{1}{5}\cancel{\cos(5\pi)} - \cancel{\cos(-\pi)} + \frac{1}{5}\cancel{\cos(-5\pi)}\right)$

$= \boxed{0}$

3. **[6 points]** Sketch the region between $y = \cos x$ and the x-axis on the interval $0 \leq x \leq \pi/2$. Find the volume of the solid which results by rotating the region around the x-axis. (*Hint. Use disks.*)

$$V = \int_0^{\pi/2} \underset{\pi}{\pi} \underset{r^2}{\cos^2 x} \, \underset{dx}{dx}$$



$$= \pi \int_0^{\pi/2} \frac{1}{2}(1 + \cos(2x))\,dx$$

$$= \frac{\pi}{2}\left[x + \frac{1}{2}\sin(2x)\right]_0^{\pi/2}$$

$$= \frac{\pi}{2}\left(\frac{\pi}{2} + \frac{1}{2}\cancel{\sin(\pi)} - 0\right) = \boxed{\frac{\pi^2}{4}}$$

EC. [1 points] (**Extra Credit**) Assume n is a large and positive integer. One of these indefinite integrals is much easier than the other. **Circle the <u>easier</u> one, and then evaluate it.**

$\int \tan^n x \sec x\,dx$ [✓ $\int \sec^n x \tan x\,dx$]

$$\int \sec^n x \tan x\,dx = \int \sec^{n-1} x \cdot \sec x \tan x\,dx$$

$$= \int u^{n-1}\,du = \frac{1}{n} u^n + c$$

$u = \sec x$

$du = \sec x \tan x$

$$= \boxed{\frac{1}{n}\sec^n x + c}$$

You may find the following **trigonometric formulas** useful. Other formulas, not listed here, should be in your memory, or you can derive them from the ones here.

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$$

$$\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$$

$$\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$$

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