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30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. [5 points] Estimate the value of $\int_0^8 x^2 dx$ using the midpoint rule with 4 sub-intervals.

2. [12 points] Compute the following improper integrals, or show that they diverge. Use appropriate limit notation for improper integrals.

a. $\int_{-\infty}^0 x e^{3x} dx =$

b. $\int_0^{\infty} \sin \theta d\theta =$

c. $\int_1^4 \frac{1}{\sqrt{4-x}} dx =$

3. [3 points] Find a formula for the n th term a_n of the sequence whose first several terms are
- $$0, 1, 3, 7, 15, 31, 63, 127, 255, \dots$$

4. [6 points] For each sequence, find the limit if it converges, or show that the sequence diverges. Indicate any places where you use L'Hôpital's rule.

a. $\frac{\sqrt{n}}{\sqrt{n} + 1}$

b. $a_n = \frac{2^n + 3^n}{4^n}$

Extra Credit. [1 point] Suppose we use the n -subinterval midpoint and trapezoid rules on an integral $\int_a^b f(x) dx$. Assume that f is continuous, and that $f''(x) > 0$ (as in Problem 1). We get results M_n and T_n , and suppose we also have the exact value E . Which of the numbers M_n, T_n, E is largest? Which number is smallest?

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