Math 252: Quiz 6 **5 March 2026**

Name: Solutions

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30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

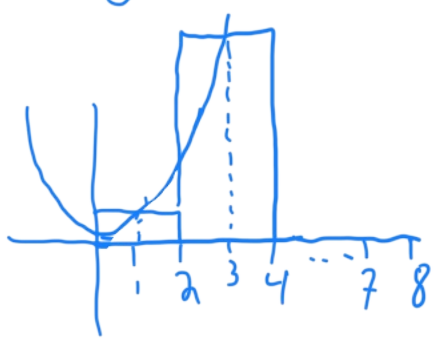
1. [5 points] Estimate the value of $\int_0^8 x^2 dx$ using the midpoint rule with 4 sub-intervals.

$$\int_0^8 x^2 dx \approx \frac{8}{4}\sum_{n=0}^{3}(2n+1)^2 = 2\left[1^2+3^2+5^2+7^2\right]$$

$$= 2\left[10+25+49\right]$$

$$= 2\left[84\right]$$

$$= \boxed{168}$$



2. [12 points] Compute the following improper integrals, or show that they diverge. *Use appropriate limit notation for improper integrals.*

$u = x \quad v = \frac{1}{3}e^{3x}$

$du = dx \quad dv = e^{3x}\,dx$

a. $\displaystyle\int_{-\infty}^{0} x e^{3x}\,dx = \lim_{b \to -\infty} \int_b^0 x e^{3x}\,dx$

$\displaystyle = \lim_{b \to -\infty} \left[\frac{x}{3} e^{3x}\right]_b^0 + \frac{1}{3}\int_b^0 e^{3x}\,dx$

$\displaystyle = \lim_{b \to -\infty} 0 - \frac{b}{3} e^{3b} + \frac{1}{9}\left[e^{3x}\right]_b^0$

$\displaystyle = 0 + \frac{1}{9} \lim_{b \to -\infty} \left(e^0 - e^{3b}\right) = \boxed{-\frac{1}{9}}$

b. $\displaystyle\int_0^{\infty} \sin\theta\,d\theta = \lim_{b \to \infty} \int_0^b \sin\theta\,d\theta = \lim_{b \to \infty} \left[-\cos\theta\right]_0^b$

$\displaystyle = \lim_{b \to \infty} -\cos b - (-\cos 0)$

$\displaystyle = 1 - \lim_{b \to \infty} \cos b$ DNE

diverges

c. $\displaystyle\int_1^4 \frac{1}{\sqrt{4-x}}\,dx = \lim_{b \to 4^-} \int_1^4 \frac{1}{\sqrt{4-x}}\,dx = \lim_{b \to 0^+} -\int_3^b u^{-1/2}\,du$

$u = 4-x$

$du = -dx$

$u(1) = 3$

$u(4) = 0$

$\displaystyle = \lim_{b \to 0^+} \int_b^3 u^{-1/2}\,du$

$\displaystyle = \lim_{b \to 0^+} \left[2\sqrt{u}\right]_b^3$

$\displaystyle = \lim_{b \to 0^+} 2\sqrt{3} - 2\sqrt{u}$

$= 2\sqrt{3} - 0$

$= \boxed{2\sqrt{3}}$

3. [3 points] Find a formula for the nth term a_n of the sequence whose first several terms are

$$0, 1, 3, 7, 15, 31, 63, 127, 255, \ldots$$

$1 + 1 = 2 = 2^1$

$3 + 1 = 4 = 2^2$

$7 + 1 = 8 = 2^3$

$15 + 1 = 16 = 2^4$

$\vdots$

$a_n + 1 = 2^n$

$$\boxed{a_n = 2^n - 1, \quad n = 0, 1, 2, \ldots}$$

$2^0 - 1 = 1 - 1 = 0$ ✓

4. [6 points] For each sequence, find the limit if it converges, or show that the sequence diverges. Indicate any places where you use L'Hôpital's rule.

a. $\dfrac{\sqrt{n}}{\sqrt{n}+1}$

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n}+1} \overset{L'H}{=} \lim_{n\to\infty} \frac{\frac{1}{2\sqrt{n}}}{\frac{1}{2\sqrt{n}}} = \boxed{1}$$

b. $a_n = \dfrac{2^n + 3^n}{4^n}$

$$\lim_{n\to\infty} \frac{2^n + 3^n}{4^n} = \lim_{n\to\infty} \left(\frac{2}{4}\right)^n + \left(\frac{3}{4}\right)^n$$

$$= \lim_{n\to\infty} \left(\frac{1}{2}\right)^n + \left(\frac{3}{4}\right)^n$$

$$= 0 + 0$$

$$= \boxed{0}$$

Extra Credit. [1 point] Suppose we use the n-subinterval midpoint and trapezoid rules on an integral $\int_a^b f(x)\,dx$. Assume that f is continuous, and that $f''(x) > 0$ (as in Problem 1). We get results M_n and T_n, and suppose we also have the exact value E. Which of the numbers M_n, T_n, E is largest? Which number is smallest?

$f''(x) > 0 \Rightarrow f$ is concave up on $[a, b]$

Midpoint underestimates

Trapezoid overestimates

$$\boxed{M_n \leq E \leq T_n}$$

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