Math 252: Quiz 7 **19 March 2026**

Name: ______________________________ ________ / 25

30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. [6 points] For each series below (i) write the series using $\sum$ notation, (ii) determine whether the series converges, (iii) explain your reasoning, (iv) if the series converges, determine its sum.

a. $1+\frac{1}{\pi}+\frac{1}{\pi^2}+\frac{1}{\pi^3}+\frac{1}{\pi^4}+\cdots$

b. $-\frac{4}{3}+\frac{16}{9}-\frac{64}{27}+\frac{256}{81}-\cdots$

2. [6 points] Consider the infinite series $\sum_{n=1}^{\infty}\left(\frac{2}{n+1}-\frac{2}{n+2}\right)$.

a. Find S_k, the kth partial sum of the series. Simplify the expression.

b. Use S_k to determine the value of series, or explain why the series diverges.

3. [2 points] State what is meant by the **harmonic series**. Does this series converge or diverge?

4. [8 points] Determine whether the series below converge or diverge. Explain your reasoning.

a. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^5}}$

b. $\sum_{n=1}^{\infty} \frac{n}{\ln(n)}$

5. [3 points] Use the Integral Test to determine whether the series $\sum_{n=1}^{\infty} ne^{-n}$ converges or diverges.

Extra Credit. [1 point] We do not know how to do the integral $\int \frac{1}{x!}\,dx$. (In fact it is not clear what "$x!$" even means for continuous values x.) However, one can argue, by comparing to a known series, that the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges. Make that argument.

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