Math 252: Quiz 7 SOLUTIONS **19 March 2026**

Name: ____________________ ________ / 25

30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. **[6 points]** For each series below (i) write the series using $\sum$ notation, (ii) determine whether the series converges, (iii) explain your reasoning, (iv) if the series converges, determine its sum.

a. $1+\frac{1}{\pi}+\frac{1}{\pi^2}+\frac{1}{\pi^3}+\frac{1}{\pi^4}+\cdots = \sum_{n=0}^{\infty}\left(\frac{1}{\pi}\right)^n$ $\left(= \sum_{n=1}^{\infty} \frac{1}{\pi^{n-1}}\right)$

geometric with $r=\frac{1}{\pi}<1$ so:

$$\sum_{n=0}^{\infty}\left(\frac{1}{\pi}\right)^n = \frac{1}{1-\frac{1}{\pi}} = \frac{\pi}{\pi-1}$$ converges

b. $-\frac{4}{3}+\frac{16}{9}-\frac{64}{27}+\frac{256}{81}-\cdots = \sum_{n=1}^{\infty}\left(-\frac{4}{3}\right)^n$

geometric with $r=-\frac{4}{3}$. but $|r|=\frac{4}{3}>1$,

so diverges

2. [6 points] Consider the infinite series $\sum_{n=1}^{\infty}\left(\frac{2}{n+1}-\frac{2}{n+2}\right)$.

a. Find S_k, the kth partial sum of the series. Simplify the expression.

$$S_k = \sum_{n=1}^{k} \frac{2}{n+1} - \frac{2}{n+2} = \frac{2}{2} - \cancel{\frac{2}{3}} + \cancel{\frac{2}{3}} - \cancel{\frac{2}{4}} + \cancel{\ldots} + \cancel{\frac{2}{k+1}} - \frac{2}{k+2}$$

$$= \boxed{1 - \frac{2}{k+2}} \quad \text{(telescoping)}$$

b. Use S_k to determine the value of series, or explain why the series diverges.

by definition:

$$\sum_{n=1}^{\infty} \frac{2}{n+1} - \frac{2}{n+2} = \lim_{k\to\infty} S_k = 1 - \lim_{k\to\infty}\frac{2}{k+2}$$

$$= 1 - 0 = 1 \quad \text{(converges)}$$

3. [2 points] State what is meant by the **harmonic series**. Does this series converge or diverge?

the harmonic series is $\sum_{n=1}^{\infty} \frac{1}{n}$.

this series diverges

4. [8 points] Determine whether the series below converge or diverge. Explain your reasoning.

a. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^5}}$ $= \sum_{n=1}^{\infty} \frac{1}{n^{5/4}}$

this converges by p-series test,

because $p > 1$

(or use integral test to show:

$$\int_1^\infty \frac{1}{x^{5/4}}\,dx = \lim_{t\to\infty} \left[-4x^{-1/4}\right]_1^t = 4 < \infty \therefore$$ converges)

b. $\sum_{n=1}^{\infty} \frac{n}{\ln(n)}$ $a_n = \frac{n}{\ln n}$

$$\lim_{n\to\infty} \frac{n}{\ln n} \overset{LH}{\underset{\frac{\infty}{\infty}}{=}} \lim_{n\to\infty} \frac{1}{\frac{1}{n}} = \lim_{n\to\infty} n = +\infty$$

since $\lim_{n\to\infty} a_n \neq 0$, diverges

by the divergence test

5. [3 points] Use the Integral Test to determine whether the series $\sum_{n=1}^{\infty} ne^{-n}$ converges or diverges.

$$\int_1^\infty x e^{-x}\,dx = \lim_{t\to\infty} \left[-xe^{-x}\right]_1^t + \int_1^t e^{-x}\,dx$$

$$\begin{bmatrix} u = x & v = -e^{-x} \\ du = dx & dv = e^{-x}dx \end{bmatrix}$$

$$= \left(\lim_{t\to\infty} -te^{-t}\right) + 1\cdot e^{-1} + \lim_{t\to\infty}\left[-e^{-x}\right]_1^t$$

$$= 0 + \frac{1}{e} - \lim_{t\to\infty} \cancelto{0}{e^{-t}} + \frac{1}{e} = \frac{2}{e}$$

$\therefore$ converges by integral test

Extra Credit. [1 point] We do not know how to do the integral $\int \frac{1}{x!}\,dx$. (In fact it is not clear what "$x!$" even means for continuous values x.) However, one can argue, by comparing to a known series, that the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges. Make that argument.

note that $n! \geq 2^n$ for $n \geq 3$. thus

$$0 \leq \sum_{n=1}^{\infty} \frac{1}{n!} = \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \sum_{n=3}^{\infty} \frac{1}{n!} \leq \frac{10}{6} + \sum_{n=3}^{\infty} \frac{1}{2^n}$$

which converges because it is geometric with $r = \frac{1}{2} < 1$

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