Math 252: Quiz 8 **26 Mar 2026**

Name: ______________________________ | ________ / 25

1. [12 points] Use the comparison test or the limit comparison test to determine if the series converges or diverges. A complete answer includes (i) which test you are using, (ii) a clear application of the test, and (iii) a conclusion drawn from the test.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^2}$

$0 \le \sin^2(n) \le 1$

$\boxed{\text{Comparison Test}}$ with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$: $\dfrac{\sin^2 n}{n^2} \le \dfrac{1}{n^2}$ for all $n \ge 1$.

Since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series with $p=2$),

$\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^2}$ also $\boxed{\text{converges.}}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-3}$

$\boxed{\text{Limit Comparison Test}}$ with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$:

$$\lim_{n\to\infty} \frac{\frac{1}{2n-3}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{2n-3} = \frac{1}{2}$$

So the two series both converge or both diverge.

Since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic series),

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-3}$ $\boxed{\text{diverges}}$ as well.

c. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{5^n - 4^n}$

$\boxed{\text{Limit Comparison test}}$ with $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{5}\right)^n$, which converges (geometric series, $r = \frac{3}{5} < 1$)

$$\lim_{n\to\infty} \frac{\frac{3^n}{5^n - 4^n}}{\frac{3^n}{5^n}} = \lim_{n\to\infty} \frac{\cancel{3^n}}{\cancel{5^n} - 4^n} \cdot \frac{\cancel{5^n}}{\cancel{3^n}}$$

$$= \lim_{n\to\infty} \frac{1}{1 - \left(\frac{4}{5}\right)^n} = 1$$

So $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{5^n - 4^n}$ $\boxed{\text{converges}}$

2. [12 points] Do the series converge absolutely, conditionally, or neither (diverge)? A complete answer includes (i) which test(s) you are using, (ii) a clear application of the test(s), and (iii) a circled answer.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2n+1}$ $\boxed{\text{Divergence test}}$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} n}{2n+1} \right| = \lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2} \neq 0,$$

So $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2n+1}$ $\boxed{\text{diverges}}$.

CONVERGES ABSOLUTELY | CONVERGES CONDITIONALLY | DIVERGES

b. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n^2}$

Test for absolute convergence : $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$

$\boxed{\text{Limit Comparison}}$ with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

$$\lim_{n\to\infty} \frac{\frac{\ln(n)}{n^2}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{\ln(n)}{n^2} \cdot n^{3/2} = \lim_{n\to\infty} \frac{\ln(n)}{n^{1/2}}$$

$$= \lim_{n\to\infty} \frac{1/n}{\frac{1}{2} n^{-1/2}} \qquad \text{(L'Hopital's rule)}$$

$$= \lim_{n\to\infty} \frac{2}{n^{1/2}}$$

$$= 0$$

Since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges (p-series, $p = 3/2 > 1$),

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n^2}$ $\boxed{\text{converges absolutely}}$

CONVERGES ABSOLUTELY

CONVERGES CONDITIONALLY

DIVERGES

c. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$

$\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n^{2/3}} \right| = \sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$ diverges (p-series, $p = \frac{2}{3} < 1$)

so $\boxed{\text{no absolute convergence.}}$

$\boxed{\text{Alternating Series test:}}$ $b_n = \frac{1}{n^{2/3}}$

$0 \le \frac{1}{(n+1)^{2/3}} < \frac{1}{n^{2/3}}$ ($f(x) = \frac{1}{x^{2/3}}$ is positive and decreasing for $x \ge 1$)

and $\lim_{n\to\infty} \frac{1}{n^{2/3}} = 0$,

So $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$ $\boxed{\text{converges conditionally}}$

CONVERGES ABSOLUTELY | CONVERGES CONDITIONALLY | DIVERGES

3. [1 points] The sum of the convergent series $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+\sqrt{n}}$ is estimated by its 15th partial sum $S_{15} = \sum_{n=1}^{15} \frac{(-1)^{n+1}}{n+\sqrt{n}} \approx 0.3523$. Estimate how close S_{15} is to the sum of the series $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+\sqrt{n}}$.

$|R_{15}| = |S - S_{15}| \le b_{16} = \frac{1}{16+\sqrt{16}} = \frac{1}{16+4} = \frac{1}{20} = \boxed{0.05}$

S_{15} is within 0.05 of S.