Math 252: Quiz 9 **2 April 2026**

Name: Solutions

______ / 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. **Show all work** and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. (12 points) Use either the ratio test or the root test as appropriate to determine whether the series converges or diverges or state that the test is inconclusive. State the test that you are using.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n!}$

Ratio Test: $\frac{5^n}{n!} \neq 0,\ n \geq 1$ ✓

$$\lim_{n\to\infty} \frac{5^{n+1}}{(n+1)!} \cdot \frac{n!}{5^n} = \lim_{n\to\infty} \frac{5(n!)}{(n+1)(n!)} = \lim_{n\to\infty} \frac{5}{n+1} = 0 < 1 \ ✓$$

By Ratio Test, $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n!}$ $\boxed{\text{converges absolutely.}}$

(b) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{3k}{k+1}\right)^k$ Root Test:

$$\lim_{k\to\infty} \sqrt[k]{\left(\frac{3k}{k+1}\right)^k} = \lim_{k\to\infty} \frac{3k}{k+1} = 3 > 1 \quad X$$

By the Root Test, $\displaystyle\sum_{k=1}^{\infty} \left(\frac{3k}{k+1}\right)^k$ $\boxed{\text{diverges.}}$

2. (4 points) For which p-values does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^p}{2^n}$ converge? Justify your conclusion.

Using Ratio Test:

$$\lim_{n\to\infty} \frac{(n+1)^p}{2^{n+1}} \cdot \frac{2^n}{n^p} = \lim_{n\to\infty} \frac{(n+1)^p}{2n^p} = \frac{1}{2} \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^p$$
$$= \frac{1}{2}(1)^p = \frac{1}{2} < 1$$

Doesn't depend on p!

Hence, $\displaystyle\sum_{n=1}^{\infty} \frac{n^p}{2^n}$ converges for all real values of p.

3. (9 points) Consider the series $\sum_{n=1}^{\infty} \frac{x^n}{n}$.

   (a) Find R, the radius of convergence of the series.

Suppose $x \neq 0$: $\frac{x^n}{n} \neq 0$ for $n \geq 1$.

Ratio Test: $\lim_{n\to\infty} \left| \frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n} \right| = \lim_{n\to\infty} \left| \frac{xn}{n+1} \right| = |x|$.

Need $|x| < 1$, so

$$\boxed{R = 1}$$

   (b) Determine the interval of convergence of the series. (Make sure to check any endpoints, if they exist.)

Need to check $x = \pm 1$.

(Harmonic series)

$x = 1$: $\sum_{n=1}^{\infty} \frac{1^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$, diverges as p-series $p = 1$.

$x = -1$: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ Alternating Series Test:

(Alternating Harmonic series)

✓ $0 \leq \frac{1}{n+1} < \frac{1}{n}$, $n \geq 1$

✓ $\lim_{n\to\infty} \frac{1}{n} = 0$

By A.S.T., $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges.

Interval of convergence: $\boxed{-1 \leq x < 1}$

Extra Credit (1 point) Suppose the series $\sum_{n=0}^{\infty} a_n(x-3)^n$ converges at $x = 2$. Can you conclude that the series converges at $x = 3.9$? Justify your conclusion.

Yes, the series does converge at $x=3.9$.
For $x=3$, the series is 0, so the series is centered at $x=3$.
Since the series also converges at $x=2$, the radius of convergence, R, is at least 1.
We have

$$|3.9-3| = 0.9 < 1 \leq R.$$