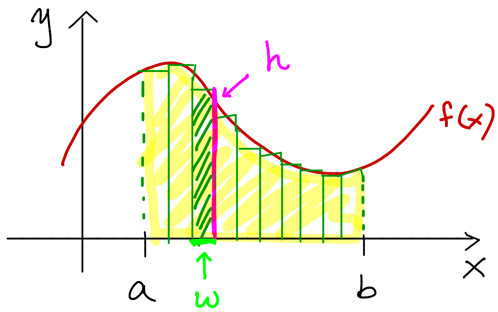
Start by showing students the cross-section demonstration and the volume by rotation demonstration.

- A Solid Defined by Cross-Sections
- Solids Defined by Rotation

1. Translating our Calculus I calculations of Area to Volumes.



Split large complicated area into $\boxed{\text{small rectangles}}$

$$\text{area rectangle} = \underbrace{h \cdot w}_{} = \overbrace{f(x)\,dx}^{}$$

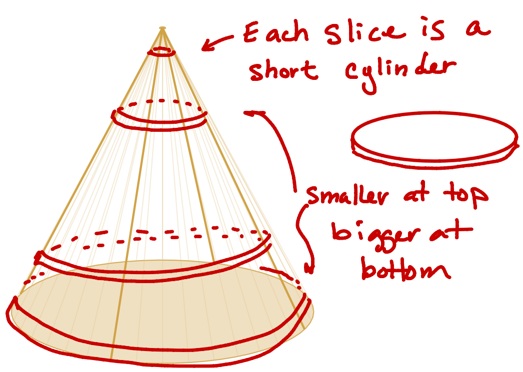
$$\text{area whole region} = A = \int_a^b f(x)\,dx$$

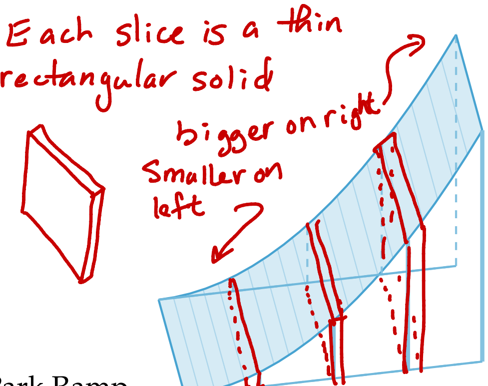
use the integral to Sum up all the tiny rectangles.

Use the same strategy to calculate the volume of complicated Solids. Instead of chopping area into tiny rectangular areas, we chop solids into tiny chunks of solid.

$$V = \int (\text{area circle})(\text{thickness})$$

$$\int (\text{area rectangle})(\text{thickness})$$



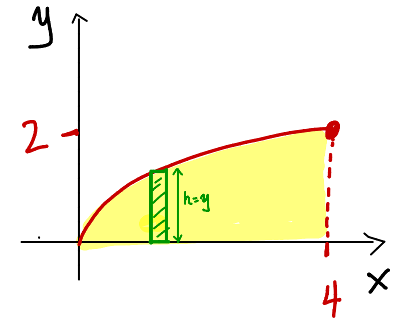


Skate Park Ramp

2. A general formula for volume using slices:

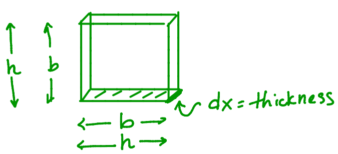
$$V = \int \underbrace{A(x)}_{\text{area of a face}}\ \underbrace{dx}_{\text{thickness of slice}}$$

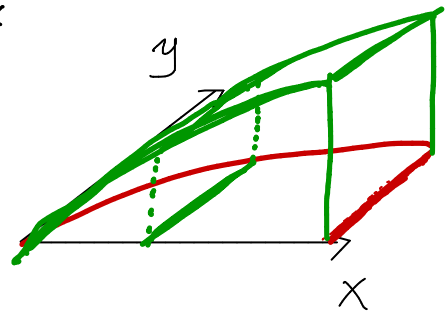
3. Sketch the region R bounded by $y=\sqrt{x}$, $y=0$, and $x=4$. Determine the volume of the solid with cross-sections perpendicular to the base and parallel to the y-axis are squares.



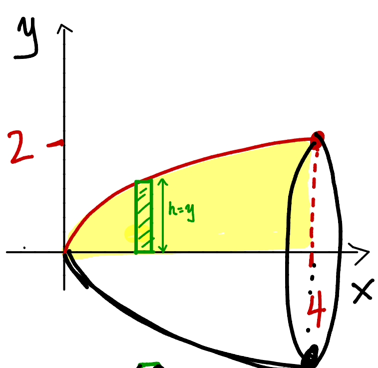
$$V=\int_0^4 \underbrace{(\sqrt{x})^2}_{\text{area of square}}\cdot dx = \int_0^4 x\,dx = \frac{1}{2}x^2\Big|_0^4$$

$$= 8 \text{ units}^3$$



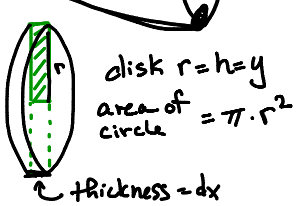


4. Sketch the same region as in problem 1 above (i.e. the region R bounded by $y=\sqrt{x}$, $y=0$, and $x=4$). Find the volume of the solid obtained by rotating this region about the x-axis. Attempt to describe and/or draw what this solid looks like.

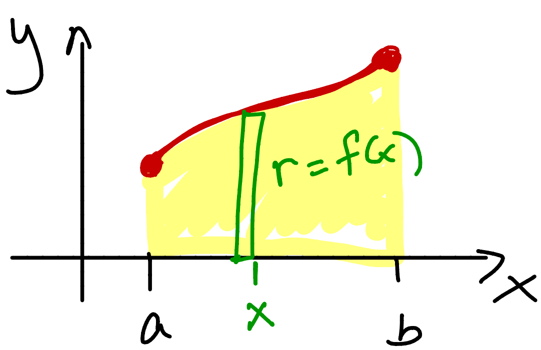


$$V=\int_0^4 \pi(\sqrt{x})^2\,dx = \int_0^4 \pi x\,dx = \frac{\pi}{2}x^2\Big]_0^4$$

$$8\pi \text{ units}^3$$

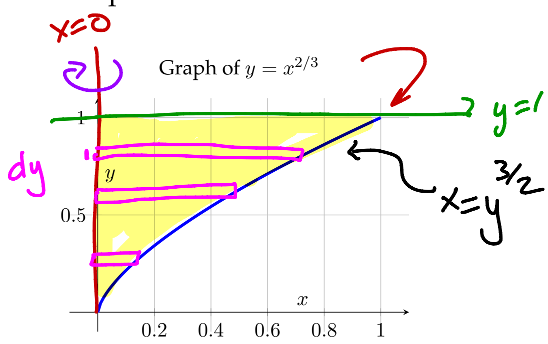


5. The Disk Method



$$V=\int_a^b \pi r^2\,dx = \int_a^b \pi (f(x))^2\,dx$$

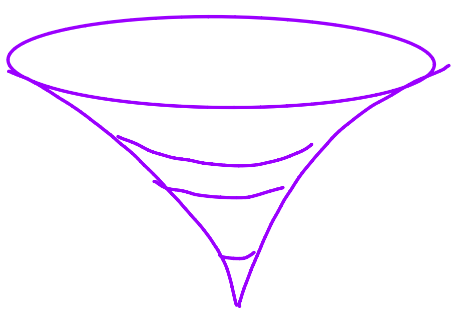
6. On the axes below, sketch and shade the region bounded by $y = x^{2/3}$, $x = 0$ and $y = 1$. Then below, sketch the solid obtained by rotating this region about the y-axis. **Set up** an integral to calculate the volume of the solid. Include your sample slice. Evaluate this integral once your have completed the rest of the sheet.



$$V = \int_0^1 \left(y^{3/2}\right)^2 dy = \int_0^1 y^3\,dy$$

$$= \frac{1}{4} y^4 \Big]_0^1 = \frac{1}{4}$$

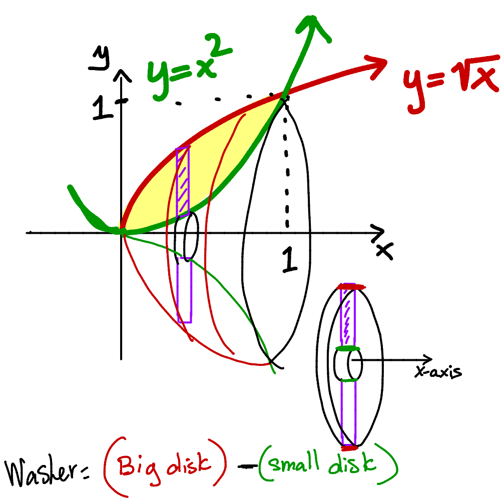
Sketch



So $y = x^{2/3}$

is $x = y^{3/2}$

7. Sketch, label, and shade the region bounded by $y = \sqrt{x}$ and $y = x^2$. In another place, sketch the solid obtained by rotating this region about the x-axis. **Set up** an integral to calculate the volume of the solid. Include your sample slice. Evaluate this integral once your have completed the rest of the sheet.



$$V = \int_0^1 \left[\pi\left(\sqrt{x}\right)^2 - \pi\left(x^2\right)^2\right] dx$$

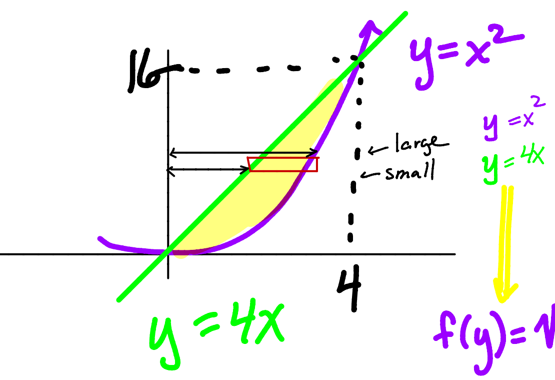
$$= \pi \int_0^1 \left(x - x^4\right) dx = \pi\left(\frac{1}{2}x - \frac{1}{5}x^5\right)\Big|_0^1$$

$$= \pi\left(\frac{1}{2} - \frac{1}{5}\right) = \frac{3\pi}{10}$$

8. The Washer Method

$$V = \int_a^b \pi \left(R^2 - r^2\right) dx = \pi \int_a^b \underbrace{[f(x)]^2}_{\text{big}} - \underbrace{[g(x)]^2}_{\text{little}} dx$$

9. Find the volume of the solid obtained by rotating about the y axis the region bounded by $y = x^2$ and $y = 4x$. (Sketch the region. Draw a slice.)



$f(y) = \sqrt{y}$

$g(y) = \frac{1}{8}y$

$$V = \int_0^{16} \pi \left(\left(\sqrt{y}\right)^2 - \left(\frac{y}{8}\right)^2 \right) dy = \pi \int_0^{16} \left(y - \frac{y^2}{64} \right) dy$$

$$= \pi \left(\frac{1}{2}y^2 - \frac{1}{192}y^3 \right)\Big|_0^{16} = \pi \left(128 - 21\tfrac{1}{3}\right)$$

$$= 106\tfrac{2}{3}\pi$$