1. Recall from 3.6

   (a) The Midpoint Rule:

$$\int_a^b f(x)\,dx = M_n = \frac{b-a}{n}\left(f(m_1)+f(m_2)+\dots+f(m_n)\right)$$

   (b) The Trapezoid Rule:

$$\int_a^b f(x)\,dx = T_n = \frac{b-a}{n}\cdot\frac{1}{2}\left(f(x_0)+2f(x_1)+2f(x_2)+\dots+2f(x_{n-1})+f(x_n)\right)$$

   (c) Simpson's Rule:

$$\int_a^b f(x)\,dx = S_n = \frac{b-a}{n}\cdot\frac{1}{3}\left(f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\dots+4f(x_{n-1})+f(x_n)\right)$$

2. We estimated $\int_0^2 e^{x^2}\,dx$ using

error (use WA)

   (a) M_4

   14.4856

$|14.4856 - 16.452627..| = 1.967$

   (b) T_4

   20.6446

$|T_4 - W| = 4.19197$

   (c) S_4

   17.3536

$|S_4 - W| = 0.90097$

   (d) Wolfram Alpha as 16.45262776550 $= W$

3. It is possible to estimate error.

(a) For the Midpoint Rule.

$$\text{error in } M_n \leq \frac{M(b-a)^3}{24n^2}$$ where $M = \max$ of $f''(x)$ on $[a,b]$

For us: $\int_0^2 e^{x^2}\,dx \approx M_4$

$n=4 \quad b=2,\ a=0$

$f(x)=e^{x^2},\ f''(x)=(4x^2+2)e^{x^2}$

$f''(2)=(4\cdot 2^2+2)e^4 < \underline{1600 = M}$

$$\text{estimate error} \leq \frac{1600(2-0)^3}{24\cdot 4^2} = 33$$ (Actually w/i 2 units)

Maybe we $\underline{\text{want}}$ to be w/ $\frac{1}{10}=0.1$. How many intervals? How big is n?

Solve $\frac{1600\cdot 8}{24\cdot n^2}=\frac{1}{10}$ or $n^2=\frac{(16{,}000)(8)}{24}$, $n=73.029$. So $n \geq 74$

(b) For Simpson's Rule.

$$\text{error in } S_n \leq \frac{M(b-a)^5}{180n^4}$$ $M = \max$ of $f^{(4)}(x)$ over $[a,b]$

$f^{(4)}(x)=(16x^4+48x^2+12)e^{x^2}$

Since $f^{(4)}(2)=25115.15 < 25116 = M$

$$\text{estimate} < \frac{(25116)(2)^5}{(180)\,4^4} = 17.4$$

Solve: $\frac{(25116)(2)^5}{180\,n^4} \leq \frac{1}{10}$ or $\frac{25116(2)^5\cdot 10}{180} \leq n^4$, $n=14.5$

So $n \geq 15$