SECTION 6.1: POWER SERIES (DAY 1)

(1) A **power series centered at** $x = 0$ has the form:

$$\sum_{n=0}^{\infty} c_n x^n = c_0 x^0 + c_1 x^1 + c_2 x^2 + c_3 x^3 + \cdots$$
$$= c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

just $a=0$!

(2) A **power series centered at** $x = a$ has the form:

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 (x-a)^0 + c_1 (x-a)^1 + c_2 (x-a)^2 + c_3 (x-a)^3 + \dots$$
$$= c_0 + c_1 (x-a) + c_2 (x-a)^2 + c_3 (x-a)^3 + \dots$$

(3) Some Examples

(a) $\sum_{n=0}^{\infty} (n+1)x^n = (0+1)x^0 + (1+1)x^1 + (2+1)x^2 + (3+1)x^3 + \dots$
$$= 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + \dots$$

Just thinking...

Pick some x-values

✓ $x=0$ $\quad \sum 0 = 0 + 0 + \dots$ converges

✗ $x=1$ $\quad \sum (n+1) = 1+2+3+4+$ diverges.

✓ $x=\frac{1}{2}$ $\quad \sum (n+1)\left(\frac{1}{2}\right)^n = 1 + 2\cdot\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \dots$

✓ $x = -\frac{1}{2}$: $\sum (n+1)\left(-\frac{1}{2}\right)^n = \sum (-1)^n (n+1)\left(\frac{1}{2}\right)^n$

For some x-values $\sum (n+1)x^n$ makes sense (converges). In others, it doesn't.

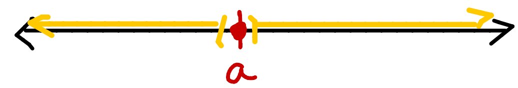
(b) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{(n+1)^4} = \frac{(x-2)^0}{(0+1)^4} + \frac{(x-2)^1}{(1+1)^4} + \frac{(x-2)^2}{(2+1)^4} + \frac{(x-2)^3}{(3+1)^4} + \dots$
$$= 1 + \frac{1}{2^4}(x-2) + \frac{1}{3^4}(x-2)^2 + \frac{1}{4^4}(x-2)^3 + \frac{1}{5^4}(x-2)^4 + \dots$$

$x=2$: $\sum 0 = 0$, converg.

$x=0$: $\sum \frac{(-2)^n}{(n+1)^4} = 1 + \frac{1}{2^4}(-2) + \frac{1}{3^4}(-2)^2 + \frac{1}{4^4}(-2)^3 + \frac{1}{5^4}(-2)^4 + \dots$

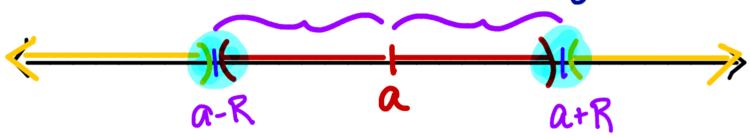
(4) Convergence of a Power Series. The convergence of $\sum_{n=0}^{\infty} c_n(x-a)^n$ looks like one of the following:

(i) it converges at $x=a$; diverges for $x \neq a$.



(ii) It converges for all real numbers

(iii) it converges for $|x-a|<R$, diverges for $|x-a|>R$ for some number R. When $|x-a|=R$, it may/maynot converge.



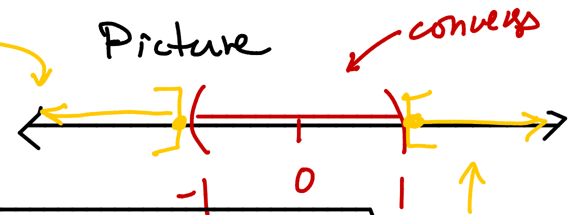
(5) Determine the **radius** and **interval** of convergence for the power series below.

(a) $\sum_{n=0}^{\infty}(n+1)x^n$

ratio test

$$\lim_{n\to\infty}\left|\frac{(n+2)x^{n+1}}{(n+1)x^n}\right| = \lim_{n\to\infty}\frac{n+2}{n+1}\cdot|x|$$

$$= |x|\cdot\lim_{n\to\infty}\frac{n+2}{n+1} = |x|\cdot 1 < 1 \text{ for convergence. So } \boxed{R=1}$$



Check endpoints

$x=1$: $\sum n+1 = 1+2+3+4+\dots$

$x=-1$: $\sum(-1)^n(n+1) = -1+2-3+4-\dots$

Diverges

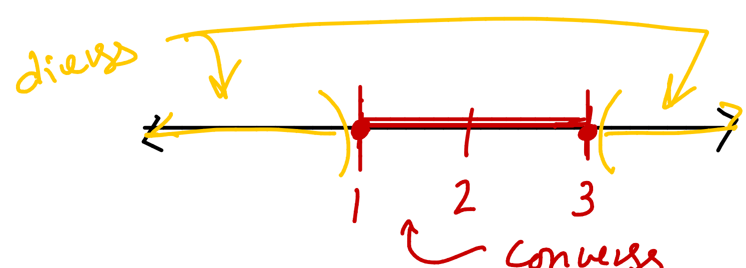
$\boxed{\text{Interval: } (-1,1)}$

(b) $\sum_{n=0}^{\infty}\frac{(x-2)^n}{(n+1)^4}$

ratio test: $\lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{(n+2)^4}\cdot\frac{(n+1)^4}{(x-2)^n}\right| = |x-2|\cdot\lim_{n\to\infty}\left(\frac{n+1}{n+2}\right)^4 = |x-2| < 1$. So $\boxed{R=1}$

So $-1 < x-2 < 1$

or $1 < x < 3$



Check endpoints:

$x=1$: $\sum\frac{1^n}{(n+1)^4} = \sum\frac{1}{(n+1)^4}$, convergent p-series

$x=3$: $\sum\frac{(-1)^n}{(n+1)^4}$, converges. absolutely

Interval: $[1,3]$