Section 6.4: Working with Taylor Series (Day 2)

XS

(1) Write the Maclaurin Series for $f(x) = \sin(x)$ and determine its interval of convergence.

$$f(x) = \sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = 1 - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

n	f	at a=0
0	$\sin x$	0
1	$f' = \cos x$	1
2	$f'' = -\sin x$	0
3	$f''' = -\cos x$	-1
4	$f^{(4)} = \sin x$	0
5	$f^{(5)} = \cos x$	1

- only odd powers of x
- start positive, then alternate

$\boxed{\text{ratio test}}$ $\lim_{n\to\infty} \left| \frac{x^{2n+3}}{(2n+3)!} \cdot \frac{(2n+1)!}{x^{2n+1}} \right|$

$$= \lim_{n\to\infty} \frac{x^2}{(2n+3)(2n+2)} = 0 < 1$$

$\boxed{\text{IOC: } (-\infty, \infty)}$

(2) Use your answer in problem 1 to find the Maclaurin series for

(a) $g(x) = x\sin(2x)$

$$= x\left(\sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+2}}{(2n+1)!}$$

(b) $h(x) = \cos(\sqrt{x})$

$$\cos(x) = \frac{d}{dx}[\sin x] = \sum_{n=0}^{\infty} \frac{(-1)^n (2n+1) x^{2n}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots \qquad \text{IOC}: (-\infty, \infty)$$

$$\cos(\sqrt{x}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (\sqrt{x})^{2n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n$$

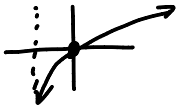
FYI: $$\int_0^1 \cos(\sqrt{x})\,dx = \sum_{n=0}^{\infty} \int_0^1 \left(\frac{(-1)^n}{(2n)!} x^n\right) dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(n+1)(2n)!}\Bigg|_0^1$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)(2n)!}$$

(3) Several Maclaurin Series to know:

(a) $f(x) = e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$ IOC $(-\infty, \infty)$

(b) $f(x) = \ln(1+x)) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$ IOC $(-1, 1]$



(c) $f(x) = \arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$ IOC $[-1, 1]$

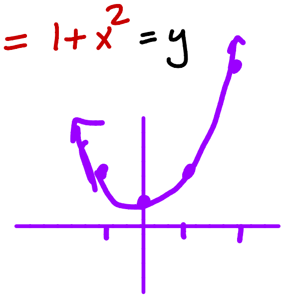
(4) We now know the sum of the alternating harmonic series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n} = \ln(2)$$

Start § 7.1 parametrically defined cures

old way $f(x) = 1 + x^2 = y$

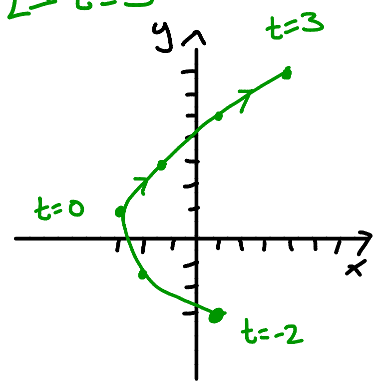
x	y
-1	2
0	1
1	2
2	5



new way: $x(t) = t^2 - 3$, $y(t) = 2t + 1$

$-2 \leq t \leq 3$

t	x	y
-2	1	-3
-1	-2	-1
0	-3	1
1	-2	3
2	1	5
3	6	7



t- parameter

- introduces directin

Eliminate the parameter

$\frac{(y-1)}{2} = t$. So $x = \left(\frac{y-1}{2}\right)^2 - 3$

or $x = \frac{1}{4}(y-1)^2 - 3$ ← parabola