

SECTION 7.1: PARAMETRIC EQUATIONS

(1) Sketch the parametric equations below. Give the orientation of the curve.

(a) $x(t) = t - 1$, $y(t) = 2t + 4$

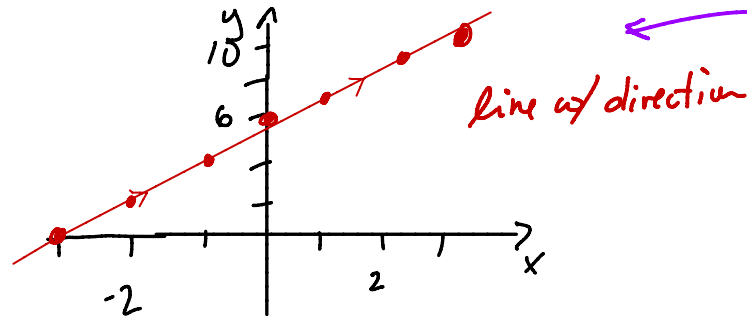
t	-2	-1	0	1	2	3
x	-3	-2	-1	0	1	2
y	0	2	4	6	8	10

$$t = x + 1$$

$$y = 2t + 4 = 2(x + 1) + 4$$

$$y = 2x + 6$$

Note: Need to pay attention to see the direction after eliminating the parameter!



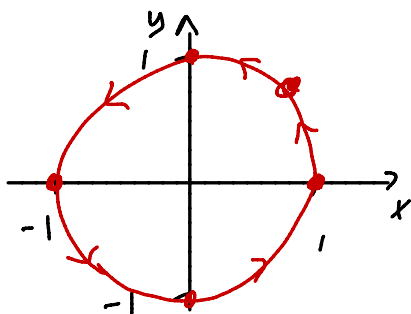
(b) $x(t) = \cos(t)$, $y(t) = \sin(t)$

t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	1	$\frac{\sqrt{2}}{2}$	0	-1	0	1
y	0	$\frac{\sqrt{2}}{2}$	1	0	-1	0

use:

$$\sin^2 t + \cos^2 t = 1$$

$$x^2 + y^2 = 1$$



(c) $x(t) = t^3$, $y(t) = 2t + 1$

t	-2	-1	0	1	2	3
x	-8	-1	0	1	8	27
y	-3	-1	1	3	5	7

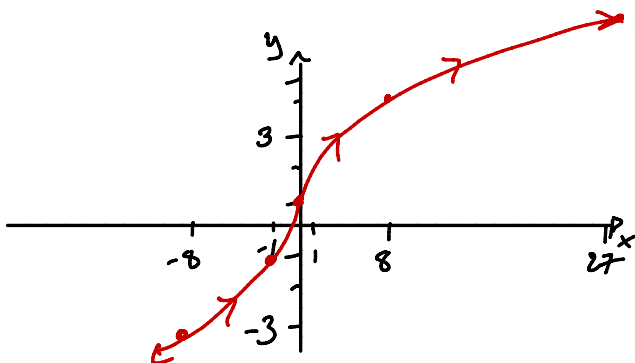
$$y = 2t + 1$$

$$\text{so } t = \frac{y-1}{2}$$

$$\text{so } x = \left(\frac{y-1}{2}\right)^3 \text{ or}$$

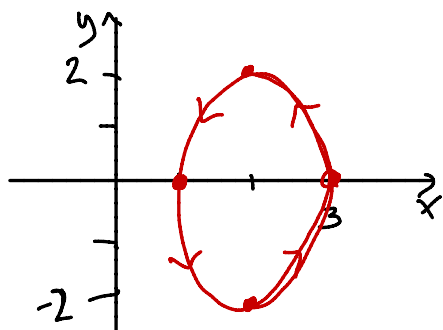
$$x(y) = \frac{1}{8}(y-1)^3$$

(cubic, shifted up 1 unit)



(d) $x(t) = 2 + \cos(t)$, $y(t) = 2 \sin(t)$

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	3	2	1	2	3
y	0	2	0	-2	0



Again use $\cos^2 t + \sin^2 t = 1$

And,

$$\cos(t) = x - 2$$

$$\sin(t) = \frac{y}{2}$$

So $(x-2)^2 + \frac{y^2}{4} = 1$ ← an ellipse.

(2) For each problem above, eliminate the parameter.

(3) Find two different ways to parametrize $y = x^2$.

$x = t, y = t^2$ $-1 \leq t \leq 1$ //
 $x = 2t, y = 4t^2$ $-\frac{1}{2} \leq t \leq \frac{1}{2}$ //
 $x = -t, y = t^2$ $-1 \leq t \leq 1$

What is the impact of different parametrizations?

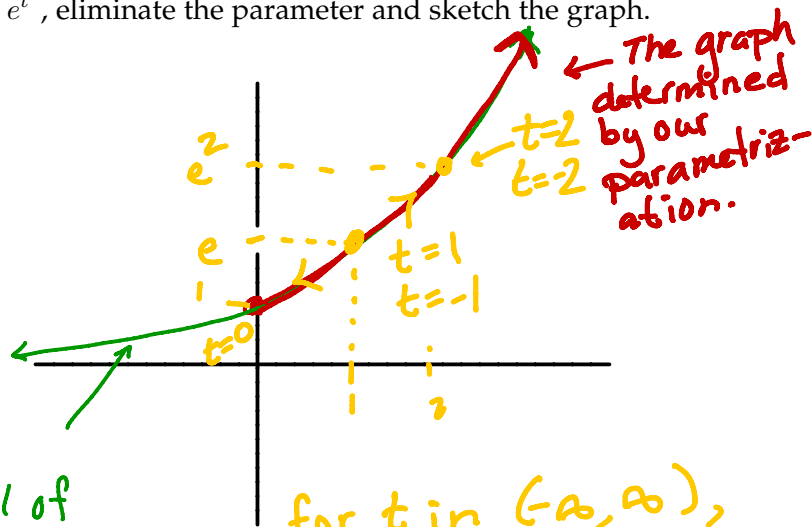
(4) For the parametric equations $x(t) = t^2$, $y(t) = e^{t^2}$, eliminate the parameter and sketch the graph. State the domain.

$$x = t^2, y = e^x$$

↑
but this parametrization

forces $x \geq 0$.

So we do not graph
all of $y = e^x$.



all of
 $y = e^x$

for t in $(-\infty, \infty)$,
the graph is followed down to
 $y=1$ then back up again.

(5) Use technology to sketch the parametric equations below.

(a) $x(t) = 1 - \sin(t)$, $y(t) = 1 - \cos(t)$

In Desmos: $(1 - \sin(t), 1 - \cos(t))$

(b) $x(t) = 3 \cos(t) + \cos(3t)$, $y(t) = 3 \sin(t) - \sin(3t)$