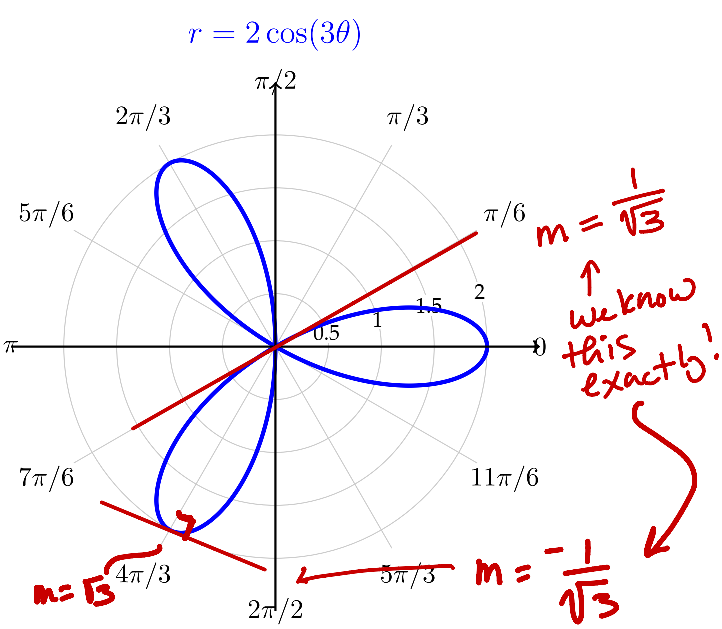
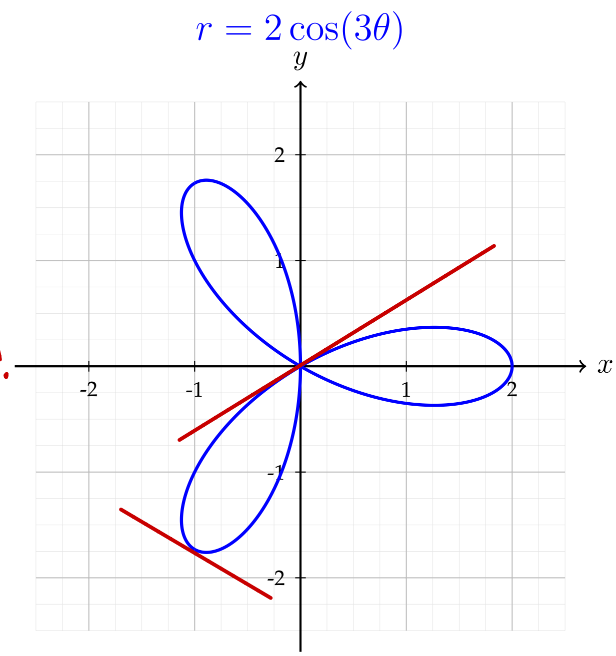
SECTION 7.4: SLOPE AND ARC LENGTH IN POLAR COORDINATES (EXTRA)

Recall the polar curve $r = f(\theta) = 2\cos(3\theta)$, graphed below.





(1) On the graphs above, sketch a tangent line at $\theta = \pi/6$ and another one at $\theta = \pi/3$. Next to your tangent lines, estimate the slope.

(2) Find $f'(\theta)$ and evaluate it at $\theta = \pi/6$ and $\theta = \pi/3$. Does this fit with your estimates above?

If $f(\theta) = 2\cos 3\theta$, then $f'(\theta) = 6\sin(3\theta)$

$f'(\frac{\pi}{6}) = 6\sin(\frac{\pi}{2}) = 6$, $f'(\frac{\pi}{3}) = 6\sin(\pi) = 0$

No. What is going on?
Here we calculated how r changes with respect to θ, NOT how y changes with respect to x!

(3) Review for the Final Exam
If we have a parametrized curve $x(t)$, $y(t)$, how do we find dy/dx?

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

(But $y = r\sin\theta = f(\theta)\sin\theta$
$x = r\cos\theta = f(\theta)\cos\theta$.

(4) Use this to find dy/dx for the polar curve and evaluate this at ~~$\theta = \pi/6$~~ and $\theta = \pi/3$.

$$\frac{dy}{dx} = \frac{f'(\theta)\sin(\theta) + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin(\theta)} = \frac{-6\sin(3\theta)\sin(\theta) + 2\cos(3\theta)\cos(\theta)}{-6\sin(3\theta)\cos(\theta) - 2\cos(3\theta)\sin(\theta)}$$

$$\left.\frac{dy}{dx}\right|_{\frac{\pi}{3}} = \frac{0 + 2\cdot(-1)(\frac{1}{2})}{0 - 2\cdot(-1)(\frac{\sqrt{3}}{2})} = \frac{-1}{\sqrt{3}} \checkmark$$

(5) Review for the Final Exam

(6) If we have a parametrized curve $x(t)$, $y(t)$, how do we find its arc length from $t = \alpha$ to $t = \beta$?

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dt$$

$$\left(f'(\theta)\sin\theta + f(\theta)\cos\theta\right)^2 + \left(f'(\theta)\cos\theta - f(\theta)\sin\theta\right)^2$$

$$= [f'(\theta)]^2 + [f(\theta)]^2$$

(7) Use this formula to set up an integral to find the arc length in one petal of the 3-petal flower $r = f(\theta) = 2\cos(3\theta)$.

$$L = 2\int_{0}^{\pi/6} \sqrt{[2\cos(3\theta)]^2 + [-6\sin(3\theta)]^2}\, d\theta$$

$$\approx 4.454$$