REVIEW FOR MIDTERM II

Basics

- Thursday 11:30am-1:00pm
- You may bring a regular-sized sheet of paper with handwritten notes on the front and the back.
- Cell phones should be either in a zippered pocket of a backpack or facedown on your desk, not in your pocket or lap. Smart watches should be in a backpack.
- You will be required to sit where there is an exam (ie spread out).
- There will be two versions of the exam.

Sections Covered

• Section 3.7 Improper Integrals

How to recognize and evaluate an improper integral. How to determine if an improper integral converges or diverges.

Practical Notes:

- The first step of understanding an improper integral is to rewrite it in terms of a limit.
- You must complete the integration and substitution prior to evaluating the limit.
- If you *formally* use the method of *u*-substitution, it is safer to resubstitute prior to evaluating the limit.

• Section 5.1 Sequences

- 1. Understand the difference between a *sequence* and a *series*.
- 2. Know how to write the terms of a sequence whether the terms are given via an explicit formula or a recursive one.
- 3. Know how to write a formula for a sequence given term-by-term (i.e. look for a pattern and generalize it).

• Section 5.2 Infinite Series

- 1. Know what is meant by *the sequence of partial sums* of a series and be able to find a few of the terms.
- 2. Know that a series converges if and only if its sequence of partial sums converges. (This is the definition of convergence for series.)
- 3. Know what a geometric series is and how to determine when it converges and when it diverges. If it converges, know to what it converges.

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- 4. Know that the **harmonic series** is and that it diverges.
- 5. Know how to identify and exploit the properties of *telescoping series*.
- Section 5.3 The Divergence and Integral Tests
 - 1. How to use the Divergence Test and know its limits.
 - 2. How to use the Integral Test.
 - 3. Know what is meant by a *p*-series and under what conditions a *p*-series converges and diverges.
 - 4. Any questions about remainders will be extra credit.
- Section 5.4 Comparison Tests
 - 1. Know **how** and when to apply the (direct) comparison test and the limit comparison test.
- Section 5.5 Alternating Series
 - 1. Know what is meant by an alternating series.
 - 2. Know how to apply the Alternating Series Test.
 - 3. Know how to estimate the remainder of a convergent alternating series when the sum of the series is estimated by a partial sum.
 - 4. Know what is meant by **absolute convergence** and **conditional convergence**.
 - 5. Know that absolute convergence implies convergence and why this fact is useful.
 - 6. Know what you must do to *show* that a series is absolutely convergent.
 - 7. Know what you must do to *show* that a series is conditionally convergent.
- Section 5.6 Ratio and Root Tests
 - 1. Know how to apply the ratio test.
 - 2. Know how to apply the root test.
 - 3. Keep in mind that for the root test, you may need to be somewhat careful about your algebra when taking limits.
 - 4. Know how to work with factorials.

Summary: You need to think about what characteristics of series suggest one test or another.

Cautionary Notes:

- You cannot ever assert a series converges or diverges without a justification.
- The only series for which the justification does *not* require the application of a formal test are geometric series, *p*-series, and the harmonic series.

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• Section 6.1 Power Series and Functions

- 1. Know what is meant by a power series centered at x = a.
- 2. Know how to find the radius of convergence and the center of convergence.
- 3. Know the power series representation of $f(x) = \frac{1}{1-x}$ and how to use it to find power series representations of other similar functions.
- Section 6.2 Properties of Power Series
 - 1. Know how to operate on power series term-by-term within their radius of convergence including addition, multiplication, differentiation and integration.
- Section 6.3 Working with Taylor Series
 - 1. Know how to find the Taylor Series of a function f(x) centered at x = a.
 - 2. Know how to find the *n*th Taylor polynomial, p_n centered at x = a.
 - 3. You should know that a Maclaurin series is a Taylor series with center a = 0.
 - 4. Remainder questions will only appear as extra credit.

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