INTEGRATION PRACTICE

Evaluate the integrals below by **first** identifying the **technique** that is appropriate. Proceed only far enough to be sure your strategy will work. Then work the problems according to **which one is hardest for you.**

trig id.

1. $\int_0^{\pi/4} \sin^2(2\theta)\, dx$ $= \frac{1}{2}\int_0^{\frac{\pi}{4}} (1-\cos(4\theta))\,d\theta = \frac{1}{2}\left(\theta - \frac{1}{4}\sin(4\theta)\right)\Big|_0^{\frac{\pi}{4}}$

$= \frac{1}{2}\left(\left[\frac{\pi}{4} - \frac{1}{4}\sin(\pi)\right] - \left(0 - \frac{1}{4}\sin(0)\right)\right)$

$= \frac{\pi}{8}$

u-sub

2. $\int x\sqrt{x^2+1}\, dx$ $= \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = \frac{1}{3}(x^2+1)^{3/2} + C$

let $u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

trig sub

3. $\int \frac{\sqrt{x^2-4}}{x}\, dx$ $= \int \frac{2\tan\theta \cdot 2\sec\theta\tan\theta\,d\theta}{2\sec\theta} = 2\int \tan^2\theta\,d\theta = 2\int (\sec^2\theta - 1)\,d\theta$

let $x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-4} = 2\tan\theta$

So $\frac{\sqrt{x^2-4}}{2} = \tan\theta$;

$= 2(\tan\theta - \theta) + C = 2\left(\frac{\sqrt{x^2-4}}{2} - \arctan\left(\frac{\sqrt{x^2-4}}{2}\right)\right) + C$

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4. $\int_2^6 \ln(t)\, dt$ $= t\ln(t)\Big]_2^6 - \int_2^6 t\cdot\frac{1}{t}\,dt = 6\ln(6) - 2\ln(2) - \int_2^6 dt$

$u = \ln(t)$ $dv = dt$

$du = \frac{1}{t}dt$ $v = t$

$= 6\ln(6) - 2\ln(2) - 4$

5. $\int \frac{2x - 3}{x^2 - 3x - 4}\, dx = \int \frac{du}{u} = \ln|u| + C = \ln|x^2 - 3x - 4| + C$

let $u = x^2 - 3x - 4$

$du = (2x - 3)\, dx$

u-sub

6. $\int \frac{6x}{x^2 - 3x - 4}\, dx = \int \frac{6x\, dx}{(x-4)(x+1)} = \int \left(\frac{24/5}{x-4} + \frac{6/5}{x+1} \right) dx$

partial fractions

$\frac{6x}{x^2-3x-4} = \frac{A}{x-4} + \frac{B}{x+1}$

$6x = A(x+1) + B(x-4)$

$x = 4: \; 24 = 5A, \; A = 24/5$

$x = -1: \; -6 = B(-5), \; B = 6/5$

$= \frac{24}{5} \ln|x-4| + \frac{6}{5} \ln|x+1| + C$

7. $\int \sin^6(x) \cos^3(x)\, dx = \int \sin^6(x) \cos^2 x \cos x\, dx$

let $u = \sin x$

$du = \cos x\, dx$

$= \int \sin^6 x (1 - \sin^2 x) \cos x\, dx = \int u^6 (1 - u^2)\, du$

$= \int (u^6 - u^8)\, du = \frac{1}{7} u^7 - \frac{1}{9} u^9 + C = \frac{1}{7} \sin^7 x - \frac{1}{9} \sin^9 x + C$

trig id.

8. $\int_0^{\pi/2} x \sin(7x)\, dx = -\frac{x}{7} \cos(7x) \Big|_0^{\frac{\pi}{2}} + \frac{1}{7} \int_0^{\pi/2} \cos(7x)\, dx$

$u = x \quad dv = \sin(7x)\, dx$

$du = dx \quad v = -\frac{1}{7} \cos(7x)$

$= \left(-\frac{\pi}{14} \cos\left(\frac{\pi}{2}\right) + 0 \cdot \cos(0) \right) + \left[\frac{1}{49} \left(\sin(7x) \right) \Big|_0^{\pi/2} \right.$

$= 0 + \left(\frac{1}{49} \right) \left[\sin \frac{7\pi}{2} - \sin 0 \right] = \frac{-1}{49}$

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