LAST OF 3.2 TRIG INTEGRALS AND 3.3 TRIG SUBSTITUTION

1. Trigonometric Identities (Fill in the set of Pythagorean Identities.)

Half-Angle/Double Angle	Sum and Difference
$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$	$\sin(ax)\cos(bx) = \frac{1}{2}(\sin((a-b)x) + \sin((a+b)x))$
$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$	$\sin(ax)\sin(bx) = \frac{1}{2}(\cos((a-b)x) - \cos((a+b)x))$
$\sin(2x) = 2\sin(x)\cos(x)$	$\cos(ax)\cos(bx) = \frac{1}{2}(\cos((a-b)x) + \cos((a+b)x))$
Pythagorean	

2. Compare the following three integrals. Which integration strategy works on which one?

(a)
$$\int \cos(x) \ dx$$

(a)
$$\int \cos(x) dx$$
 (b) $\int \cos^2(x) dx$

(c)
$$\int \cos^3(x) \, dx$$

3. Compare the following three integrals. Which ones can a Calculus I student integrate?

(a)
$$\int \frac{1}{\sqrt{9-x^2}} dx$$
 (b)
$$\int \frac{x}{\sqrt{9-x^2}} dx$$

(b)
$$\int \frac{x}{\sqrt{9-x^2}} \, dx$$

$$\text{(c)} \int \frac{x^2}{\sqrt{9-x^2}} \, dx$$

4. Summary: If $\sqrt{a^2-x^2}$ appears in an integrand (and other techniques do not work), then

2 §3.2 & 3.3 5. Compare the following integrals. Which one can a Calculus I student integrate? (a) $\int \frac{dx}{9+x^2}$ (b) $\int \frac{dx}{\sqrt{9+x^2}}$ (c) $\int \frac{dx}{\sqrt{x^2-9}} \, dx$

(a)
$$\int \frac{dx}{9+x^2}$$

(b)
$$\int \frac{dx}{\sqrt{9+x^2}}$$

(c)
$$\int \frac{dx}{\sqrt{x^2 - 9}} dx$$

6. Summary:

- $\bullet\,$ If $\sqrt{a^2+x^2}$ appears in an integrand (and other techniques do not work), then
- $\bullet \ \ \mbox{If } \sqrt{x^2-a^2}$ appears in an integrand (and other techniques do not work), then

EXTRA PRACTICE

7. Evaluate

(a)
$$\int \frac{dx}{(4+x^2)^2}$$

(b)
$$\int \frac{dx}{(x^2-9)^{3/2}}$$