SOLUTIONS

Worksheet: Improper integrals! Please work in groups.

Start by finding your group. The problems are mostly improper integrals, with a bit of other stuff. For the improper integrals, use proper limit notation. Each problem has a clean simplified answer, or "diverges"; please put that answer in the box.

1. $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx =$ $\lim_{t\to\infty} \int_1^t x^{-2}\,dx = \lim_{t\to\infty} \left[-x^{-1}\right]_1^t$

$= \lim_{t\to\infty} 1 - \frac{1}{t} = 1 - 0$

1

2. $\displaystyle\int_{-\infty}^2 \cos(x)\,dx =$ $\lim_{t\to-\infty} \int_t^2 \cos(x)\,dx = \lim_{t\to-\infty} \left[\sin(x)\right]_t^2$

$= \lim_{t\to-\infty} \sin(2) - \sin(t)$

but $\lim_{t\to-\infty} \sin(t)$ d.n.e.

diverges

3. $\displaystyle\int_3^\infty xe^{-x}\,dx =$ $\lim_{t\to\infty} \left[-xe^{-x}\right]_3^t + \int_3^t e^{-x}\,dx$

$[u=x \quad v=-e^{-x}; \; du=dx \quad dv=e^{-x}dx]$ $= \lim_{t\to\infty} 3e^{-3} - te^{-t} - \left[e^{-x}\right]_3^t$

$= \lim_{t\to\infty} 3e^{-3} - te^{-t} - e^{-t} + e^{-3}$

$= 3e^{-3} - 0 - 0 + e^{-3}$

$4e^{-3}$

4. Compute the trapezoid rule approximation T_4 for the integral $\displaystyle\int_1^2 \frac{1}{x^2}\,dx$.

$\Delta x = \frac{2-1}{4} = \frac{1}{4}$, $x_0 = 1$, $x_1 = \frac{5}{4}$, $x_2 = \frac{3}{2}$, $x_3 = \frac{7}{4}$, $x_4 = 2$

$T_4 = \frac{1/4}{2}\left(\frac{1}{1^2} + 2\cdot\frac{1}{(5/4)^2} + 2\cdot\frac{1}{(3/2)^2} + 2\cdot\frac{1}{(7/4)^2} + \frac{1}{2^2}\right)$

$= 0.5090$

0.5090

5. $\int_0^1 \frac{1}{x^2}\,dx =$ $\lim_{a\to 0^+} \int_a^1 x^{-2}\,dx = \lim_{a\to 0^+} \left[-x^{-1}\right]_a^1$

$= -1 + \lim_{a\to 0^+} \frac{1}{a}$, but $\lim_{a\to 0^+} \frac{1}{a} = +\infty$

diverges

6. $\int_0^4 \frac{1}{\sqrt{x}}\,dx =$ $\lim_{a\to 0^+} \int_a^4 x^{-1/2}\,dx = \lim_{a\to 0^+} \left[2x^{\frac{1}{2}}\right]_a^4$

$= 2\cdot 4^{1/2} - \lim_{a\to 0^+} 2\cdot a^{1/2}$

$= 2\cdot 2 - 2\lim_{a\to 0^+} \sqrt{a} = 4 - 0$

4

7. $\int x^2 \cos(x^3)\,dx =$

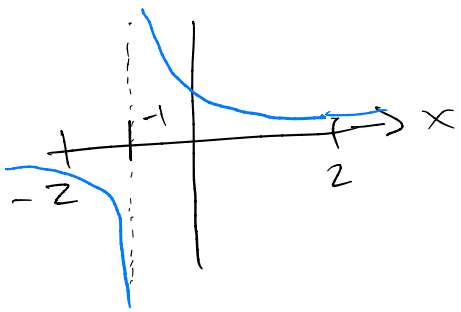
$\left[\begin{array}{l} u = x^3 \\ du/3 = x^2\,dx \end{array}\right]$

$\int \cos(u)\,\frac{du}{3} = \frac{1}{3}\sin(u) + C$

$\frac{1}{3}\sin(x^3) + C$

8. $\int_{-2}^2 \frac{1}{1+x}\,dx =$ $\int_{-2}^{-1} \frac{1}{1+x}\,dx + \int_{-1}^{2} \frac{1}{1+x}\,dx$ ← both diverge … just show for one …

graph has vertical asymptote!



$\int_{-1}^{2} \frac{1}{1+x}\,dx = \int_0^3 \frac{1}{u}\,du$ $\left[\begin{array}{l} u = 1+x \\ du = dx \end{array}\right]$

$= \lim_{a\to 0^+} \left[\ln u\right]_a^3$

$= \ln 3 - \lim_{a\to 0^+} \ln a$ ← $= -\infty$

diverges