SOLUTIONS

SECTION 5.5: ALTERNATING SERIES

1. The Alternating Series Test (AST)

A series $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$ or $\sum_{n=1}^{\infty}(-1)^n b_n$

converges if: ① $b_n \geq 0$

② b_n decreasing $(b_{n+1} \leq b_n)$

③ $\lim_{n\to\infty} b_n = 0$.

2. Determine whether the alternating series below converge or diverge. Justify your conclusion by checking the requirements of the AST.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}n}{3n+1}$

$b_n = \dfrac{n}{3n+1}$

$b_n \geq 0$ ✓

$\lim_{n\to\infty} b_n \overset{LH}{=} \lim_{n\to\infty} \dfrac{1}{3} = \dfrac{1}{3} \neq 0$

diverges by divergence test

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n}n}{2^n}$

$b_n = \dfrac{n}{2^n}$

$b_n \geq 0$

b_n decreases

$\lim_{n\to\infty} \dfrac{n}{2^n} \overset{LH}{=} \lim_{n\to\infty} \dfrac{1}{(\ln 2)2^n} = 0$

converges by AST

(c) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^{n-1}}{\ln(\ln n)}$

$b_n = \dfrac{1}{\ln(\ln n)}$

$b_n \geq 0$

b_n decrease ($\ln(\ln n)$ increases)

$\lim_{n\to\infty} \dfrac{1}{\ln(\ln n)} = 0$

$\dfrac{1}{\infty}$

converges by AST

3. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$.

(a) Show the series converges. (Justify by checking a test.) We will call the sum S.

$b_n = \frac{1}{n^2}$ $b_n \geq 0$, b_n decreases, $\lim_{n\to\infty} b_n = 0$

converges by AST

(b) Show the series converges absolutely. (Justify by checking a test.)

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ is $p=2>1$ p-series so converges

(c) Find S_3. Estimate $R_3 = S - S_3$ using the idea of alternating series remainders.

$S_3 = +\frac{1}{1} - \frac{1}{2^2} + \frac{1}{3^2} = 1 - \frac{1}{4} + \frac{1}{9} = \frac{31}{36}$

$|R_3| = |S - S_3| \leq b_4 = \frac{1}{4^2} = \frac{1}{16}$

(d) Determine how large k needs to be so that the kth partial sum, S_k, is within $0.0001 = 10^{-4}$ of the sum of the series?

$|R_k| \leq b_{k+1} = \frac{1}{(k+1)^2} \leq 10^{-4} \iff k+1 \geq 10^2$

$\iff k \geq 101$

4. For each series below, determine if the series is absolutely convergent, conditionally convergent, or divergent.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$ $b_n = \frac{1}{2n+1}$ $b_n \geq 0$, b_n decreasing, $\lim_{n\to\infty} b_n = 0$

converges by AST. but $\sum_{n=1}^{\infty} \frac{1}{2n+1}$ diverges

conditionally convergent (LCT to harmonic)

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n-1)!}{n!}$ $b_n = \frac{(n-1)!}{n!} = \frac{(n-1)!}{n(n-1)!} = \frac{1}{n}$

converges by AST, not abs. convergent ($\sum b_n$ harmonic)

conditionally convergent