

Lecture Notes: Weighted Voting, Intro + Terminology

Math F113X Math and Society

Context for Weighted Voting

Examples

- A company. Shareholder votes are proportional to the # of shares
- A HOA: weights = longevity
- US House of representatives: states get votes proportional to population

Notation and Terminology for Weighted Voting

- players

people / entities that are voting

$P_1, P_2, P_3 \dots$

- weight of a player

of votes that player gets

Player P_i has weight w_i [P_i has weight w_i]

- total weight of the voting system

= Sum of all the weights

Notation and Terminology for Weighted Voting

- quota

Minimum # of votes needed to approve an action

* Set for each situation!

- coalition

A group of players that vote together

Notation and Terminology for Weighted Voting

- winning coalition

A coalition whose weights sum to the quota, or higher!

- critical player

A player whose removal from a winning coalition means the coalition is no longer winning

Notation for a Weighted Voting System

P_1 P_2 P_3 P_4
 $[25: 11, 11, 10, 8]$
 ↑
 QUOTA
 ↓
 4 players

Total Weight = $11 + 11 + 10 + 8 = 40$

Look at some coalitions

P_1 P_2 P_3 P_4 ← players

 11 11 10 8 ← weights

In general:
 [quota: list of weights]

Coalition	weight	winning?
P_1, P_2	$11 + 11 = 22$	No! $22 < 25$
P_1, P_2, P_3	$11 + 11 + 10 = 32$	Yes! $32 \geq 25$ ← everyone is critical!
P_1, P_3, P_4	$11 + 10 + 8 = 29$	Yes! $29 \geq 25$
P_1, P_2, P_3, P_4	40	Yes! $40 \geq 25$ ← No one is critical

Reasonable Limits on the Quota

Let $q = \text{quota}$ and $T = \text{total weight}$
 $= w_1 + w_2 + \dots$

① $q \leq T$ (otherwise we can never pass anything!)

② $q > \frac{T}{2}$ (Want to require majority support)

A Look at Power

A Dictator [25: ^{P₂}25, ^{P₃}10, ^{P₄}3, 2], total weight = 40

↳ Someone who can do an action alone!
A player whose weight $w_i \geq q$

What is player 1's weight?

$$w_1 = 40 - 10 - 3 - 2 = 25$$

Notice: P_1 is a dictator!

A Look at Power

Having Veto Power

[25: 21, 11, 5, 3], total weight = 40

Veto power a member of every winning coalition

Is there a winning coalition that excludes player 1?

$$\{P_2, P_3, P_4\} : \text{weight} = 11 + 5 + 3 = 19 < 25$$

↓
Not winning!

Veto power: the remaining players do not form a winning coalition

A Look at Power

A Dummy [25: 20, 18, 1, 1], total weight = 40

A player who is never critical / removal from a coalition and it still wins

P_3, P_4 are dummies

$\{P_1, P_2\}$ weight = $20 + 18 = 28 \geq 25$ so winning

$\{P_1, P_2, \underline{P_3}\}$ ← still winning

No Dictators!

P_1, P_2 both have veto power