

Lecture Notes: Fair Division, Intro + Divider-Chooser

Math F113X Math and Society

The Context of Fair Division

People/entities are dividing up stuff

- Company dividing assets
- Heirs dividing an estate
- roommates dividing chores

* Different people value things differently!

* Assumption: Individuals can assign a fixed value to the stuff, and can make rational choices based on that value

Not allowed: I don't value X, but you can't have it.

The Definition of a Fair Share

Try #1: If there are N parties, a fair share should be $1/N$ of the total value

Refine...

Try #2: A fair share for a specific party has value of at least $1/N$ of the total value to that specific party (in other words, based on that party's own valuations)

Divider-Chooser in a Nutshell

* Two parties. "You split, I choose"

- Assign one party as the Divider and the other as Chooser
- Divider splits stuff into two equal shares S_1 & S_2 according to the divider's valuation
- The chooser picks their preferred share
- The divider gets the other share

Example Dividing Muffins

What is being divided:

6 muffins

2 apple-walnut (A)

2 blueberry (B)

2 cheese & jalepeno (C)

Cost:

the package of 6 cost \$12

Parties (who is involved):

Yuri (Y), and Zariah (Z)

Preferences:

Y likes all the flavors equally.

Z likes A twice as much as B or C

Example Dividing Muffins

2 apple-walnut (A)
2 blueberry (B)
2 cheese & jalepeno (C)

the package of 6 cost \$12

- (a) Ignoring all preferences, what is the value of a muffin?

\$2

$$\frac{\$12}{6 \text{ muffins}} = \$2/\text{muffin}$$

- (b) In a dollar amount, what would be the value of a fair share in this case? _____

$$\frac{\text{total value}}{\# \text{ people}} = \frac{\$12}{2} = \$6 = \text{value of a fair share}$$

Example Dividing Muffins

Preferences: Y likes all the flavors equally.
Z likes A twice as much as B or C

- (c) Fill out the table below indicating for each party (X,Y, or Z), the dollar amount **they** would assign to each muffin. The total value should always sum to \$12. (!!)

Method 1:
Algebra

Let $x =$
value of each B.

$$2A + 2B + 2C = 12$$

$$2(2x) + 2x + 2x = 12$$

$$8x = 12$$

$$x = \frac{12}{8} = \frac{3}{2} = 1.50$$

Method #2:
guess & check

party	A	A	B	B	C	C	total value
Y	\$2	\$2	\$2	\$2	\$2	\$2	$6(2) = 12$ ✓
Z	2	2	1	1	1	1	8 = total value
	3	3	1.50	1.50	1.50	1.50	12 = total value

*TRY#1

multiply

Method #3: Proportion

$$\frac{\text{total value}}{\text{guessed total}} = \frac{12}{8} = 1.5$$

our multiplier

Example Dividing Muffins

(d) Complete Divider-Chooser with Yuri as the divider and Zariah as the chooser.

party	A	A	B	B	C	C
Y	2	2	2	2	2	2
Z	3	3	1.5	1.5	1.5	1.5

• Z chooses S_1
because it is fair to her
(worth at least \$6)

• Y takes S_2 which is a
fair share to him!

	Shares	Yuri's value	Zariah's Value
S_1	A A B	$3(2) = \$6$	$2(3) + 1(1.5) = 7.50$ ← fair share for Z
S_2	B C C	$3(2) = \$6$	$3(1.5) = 4.50$ ← <u>NOT</u> fair to Z

Example Dividing Muffins

(e) Complete Divider-Chooser with Zariah as the divider and Yuri as the chooser.

party	A	A	B	B	C	C
Y	2	2	2	2	2	2
Z	3	3	1.5	1.5	1.5	1.5

Yuri chooses
 $BBCC = S_2$
 Zariah gets S_1

	Shares	Value to Zariah	Value to Yuri
S_1	A A	$2(3) = 6$	4
S_2	B B C C	$4(1.5) = 6$	8

← fair share to Yuri

Each party gets a fair share according to their valuation!