

Lecture Notes: Scheduling, Introduction to Scheduling

Math F113X Math and Society

Goal:

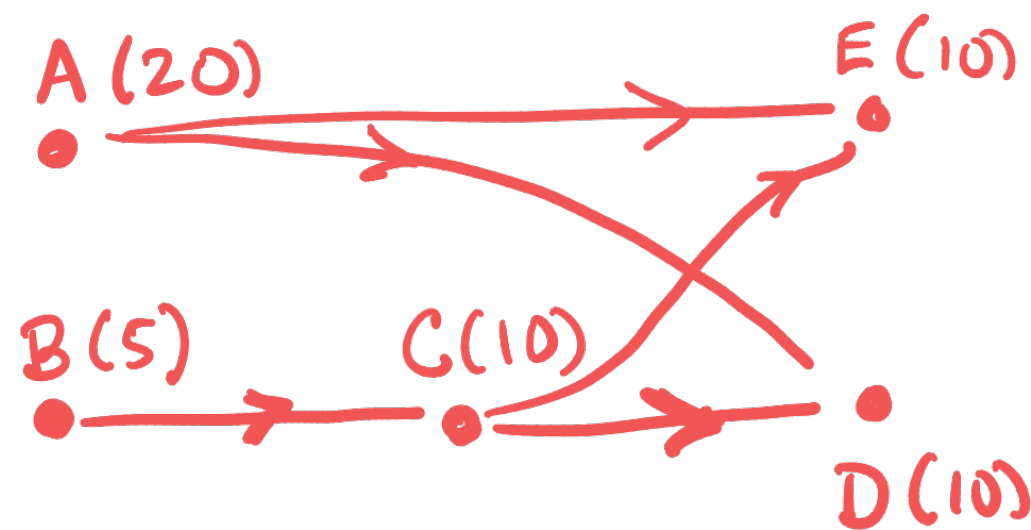
Learn about the following terminology: schedule, digraph, processors, finishing time, optimal finishing time, optimal schedule, idle time, critical path, critical time.

Real World Example: Carnegie Mellon Road Construction Plan

https://www.cmu.edu/cee/projects/PMbook/09_Construction_Planning.html/#9.3%20Defining%20Work%20Tasks

Example: Fixing a Flat Bike Tire

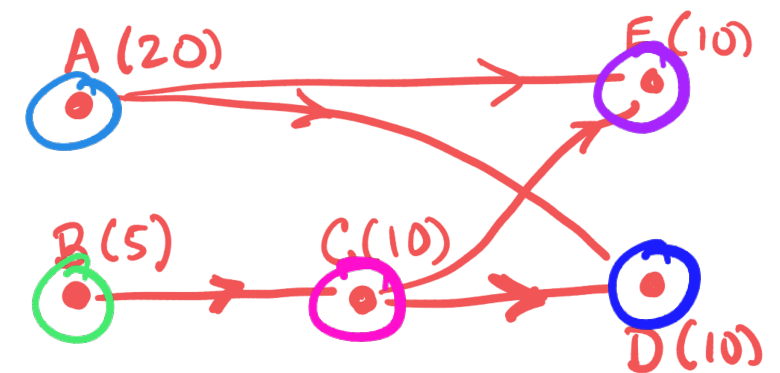
label	task	time	dependence
A	buy a new tube and patch kit	20 minutes	—
B	find tools	5 minutes	—
C	remove tire and old tube	10 minutes	B
D	insert new tube, reattach tire	10 minutes	A B C
E	repair old tube	10 minutes	A B C



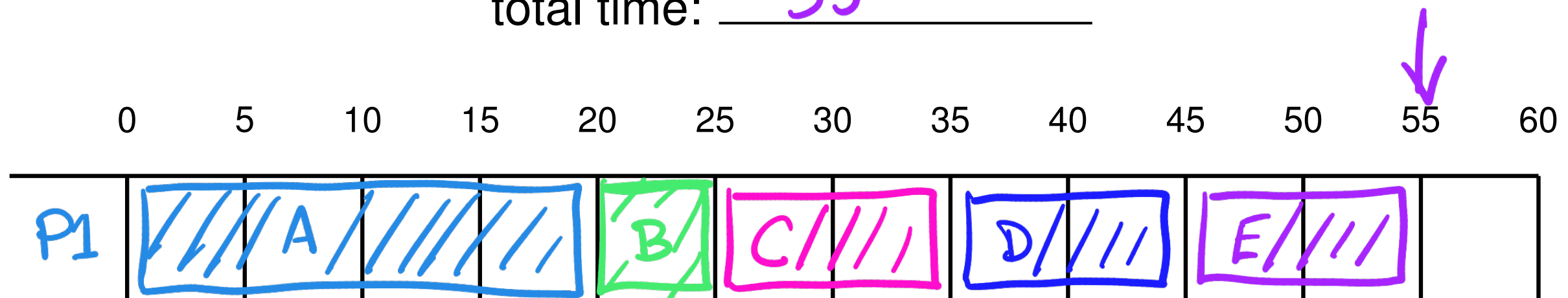
← digraph
= directed graph

Schedule with one processor

label	task	time	dependence
A	buy a new tube and patch kit	20 minutes	—
B	find tools	5 minutes	—
C	remove tire and old tube	10 minutes	B
D	insert new tube, reattach tire	10 minutes	A B C
E	repair old tube	10 minutes	A B C



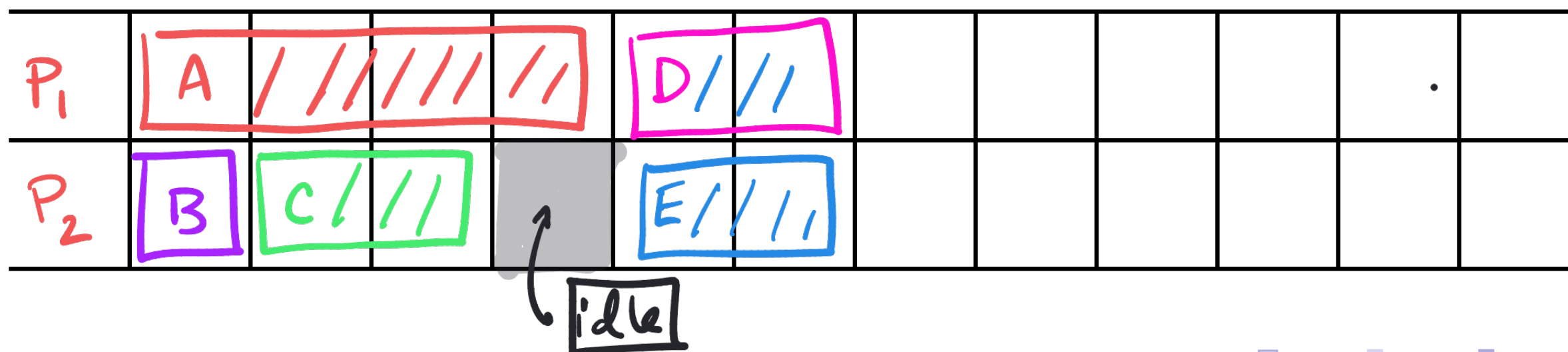
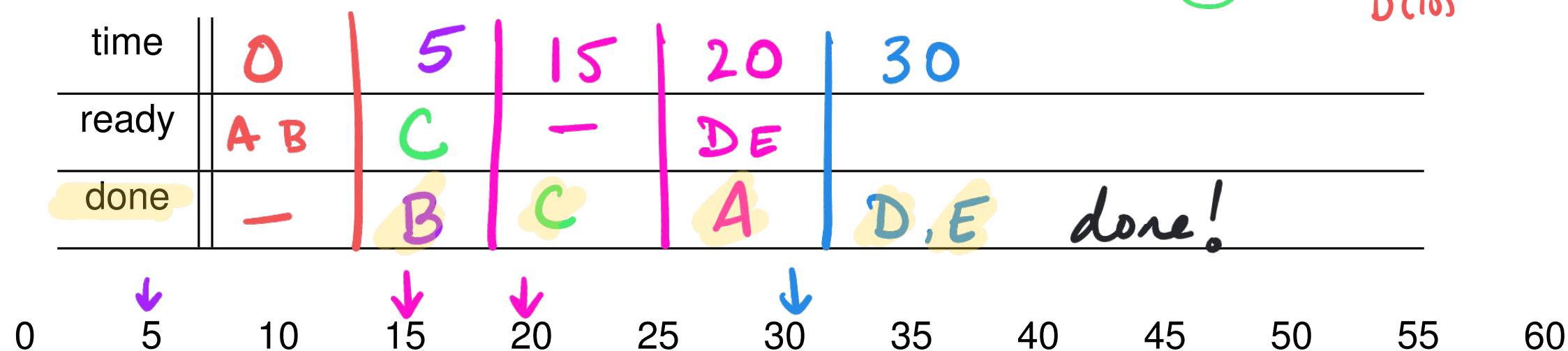
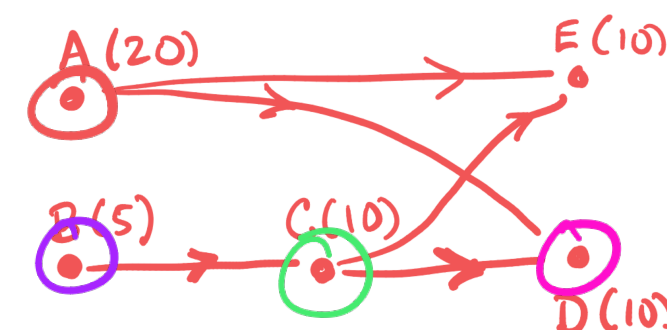
total time: 55



Schedule with two processors

task	A	B	C	D	E
time	20	5	10	10	10
dependence	—	—	B	ABC	ABC

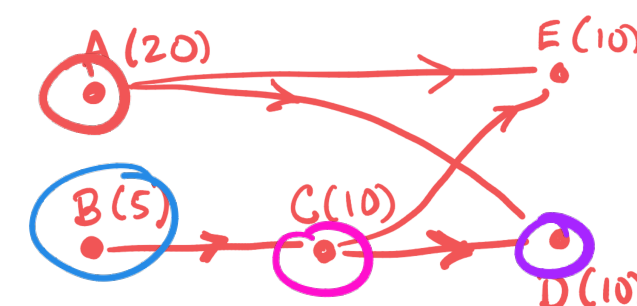
total time: 30



Schedule with three processors

task	A	B	C	D	E
time	20	5	10	10	10
dependence	—	—	B	ABC	ABC

total time: 30



time	0	5	15	20	30							
ready	A B	C		D E								
done	—	B	C	A	D E	← done						

P ₁	A / / / / / / /	D / / /										
P ₂	B	C / / / /		E / / /								
P ₃	IDLE											

Terminology

(a) **schedule**

The ordering of all tasks, accounting for dependencies

(b) **digraph**

Model of the schedule

vertices = tasks (labeled with time)

directed edges $V \longrightarrow W$

You can't complete W until you've completed V

Terminology

(c) **processors**

people / computers / teams completing tasks

(d) **finishing time**

How long it takes to finish the project/
complete all tasks

Terminology

(e) **optimal finishing time and optimal schedule**

Shortest possible finishing time
and the corresponding schedule assigning tasks
to processors

(f) **idle time**

time when processors are not completing tasks

Terminology

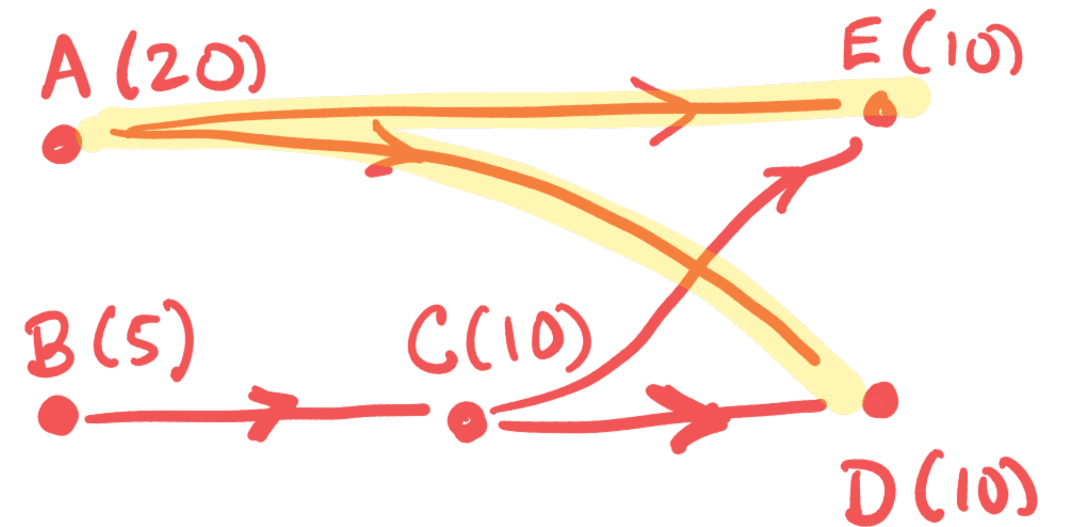
(g) critical path

Length of longest path according to the sum of the vertex weights

(h) critical time

The sum of the vertex weights on the critical path

* optimal time must be at least as long as the critical time!



Paths:

$A \rightarrow E$	30
$A \rightarrow D$	30
$B \rightarrow C \rightarrow E$	25
$B \rightarrow C \rightarrow D$	25

General Example

label/task	time	dependence
A ✓	2	
B ✓	1	
C ✓	2	
D ✓	3	(A)
E ✓	6	A, B
F ✓	8	B, C
G ✓	5	<u>E</u> , <u>F</u>

Make a valid schedule with TWO processors. Determine values of finishing time, idle time and critical time.

