

Note that before Fall 2026, Exam 1 covered Voting Theory, Weighted Voting, and Fair Division. This Exam 1 review material covers Voting Theory and Weighted Voting only.

1 Voting Theory

1.1 You should know

Terminology: majority, plurality, Cordorcet Winner

Voting Methods by Name: Plurality Method, Instant Run-off Voting (IRV) (also known as Ranked Choice Voting), Borda Count, Copeland's Method

Fairness Criteria by Name: Condorcet Criterion, Monotonicity Criterion, Majority Criterion, Irrelevance of Independent Alternatives (IIA) Criterion

1.2 Sample Voting Theory Problems

1. A class of middle schoolers are trying to decide what would be the worst ingredient to add to a cake their Principal has to eat if they win the Science Bowl. They narrow their options down to three: pickled onions (PO), stinky cheese (SC), or anchovies (A). The class is polled and the resulting preference schedule is below.

	22	5	23	2	11	20
1st choice	PO	PO	SC	SC	A	A
2nd choice	SC	A	PO	A	PO	SC
3rd choice	A	SC	A	PO	SC	PO

- (a) How many students voted? _____
- (b) How many votes does a candidate need to win in order to win a majority? Show the calculation that gives your answer. _____
- (c) How many votes does a candidate need to win in order to win a plurality? Show the calculation that gives your answer. _____
- (d) Find the winner using the plurality method. Show your work.
- (e) Find the winner using Instant Runoff Voting. Show your work.

For reference, here's the preference schedule again.

	22	5	23	2	11	20
1st choice	PO	PO	SC	SC	A	A
2nd choice	SC	A	PO	A	PO	SC
3rd choice	A	SC	A	PO	SC	PO

(f) Find the winner using Borda Count. Show your work.

(g) Find the winner using Copeland's Method. Show your work.

(h) Is any candidate a Condorcet Winner? Explain your answer.

2. Consider the following preference schedule, for an election with five candidates: Alfred, Bethanne, Carol, David, and Edgar.

Rank	40	8	25	15	5	7
1st	A	B	C	D	E	C
2nd	C	A	A	A	A	B
3rd	B	C	B	B	B	A
4th	D	D	D	C	C	D
5th	E	E	E	E	D	E

Find the winner under Instant Runoff Voting. Clearly show your rounds, and state who is eliminated in each round. For each round, clearly indicate the vote total for each candidate.

The winner is _____ and they won in _____ rounds.

3. Consider the following preference schedule, and the points assigned under Borda count. Explain which fairness criterion this shows Borda Count fails, and why this particular election fails that fairness criterion.

Rank	51	30	19	Candidate	Borda Count Points
1st	A	B	C	A	253
2nd	B	C	D	B	311
3rd	C	D	B	C	268
4th	D	A	A	D	168

4. For each of the following fairness criteria, describe what an election would look like that **failed** that criterion. (You do not need to provide an actual preference schedule, just talk about what the problem would be.)

(a) Condorcet Criterion

(b) Monotonicity Criterion

(c) Majority Criterion

(d) Irrelevance of Independent Alternatives (IIA) Criterion

2 Weighted Voting

2.1 Terminology

quota, weight, winning coalition, critical player in a winning coalition, dictator, veto power, dummy, Banzhaf Power Index or Banzhaf Power Distribution.

2.2 Sample Weighted Voting Problems

1. Nova Robotics Inc. has five board members who vote on major company decisions (funding rounds, executive hires, acquisitions, etc.). Each member's voting weight is based on the percentage of company shares they control, and company bylaws require any motion to pass with at least 60% of the total votes.

Founder/CEO: 56 shares

Co-founder/CTO: 44 shares

Lead Venture Capital firm: 40 shares

Early Angel Investor: 36 shares

Employee Stock Pool Representative: 24 shares

(a) Write a weighted voting system of the form $[q : w_1, \dots, w_n]$ that represents this scenario.

(b) Find a winning coalition in this system, and write a sentence explaining how you know it is a winning coalition.

2. For each weighted voting system below, determine if there are any dictators, anyone with veto power, or any dummies.

(a) $[16 : 16, 11, 3, 1]$

(b) $[51 : 40, 30, 20, 10]$

(c) $[31 : 10, 9, 8, 7, 6]$

3. Consider the weighted voting system $[q : 21, 20, 7, 5]$ where the players can pass a motion with a **majority**.

(a) What is q in this case? How do you know?

(b) List all winning coalitions.

(c) In each of the winning coalitions above, indicate who is a critical player.

(d) Calculate the Banzhaf Power Index for each player. Give your answer as a fraction and as a percent.

(e) Based on your calculations above, are there any dummies? Explain.